

# Mathematica 11.3 Integration Test Results

Test results for the 385 problems in "5.3.6 Exponentials of inverse tangent.m"

Problem 15: Result more than twice size of optimal antiderivative.

$$\int \frac{e^{2i \operatorname{ArcTan}[ax]}}{x} dx$$

Optimal (type 3, 13 leaves, 3 steps):

$$\operatorname{Log}[x] - 2 \operatorname{Log}[i + ax]$$

Result (type 3, 29 leaves):

$$\operatorname{Log}[1 - e^{2i \operatorname{ArcTan}[ax]}] + \operatorname{Log}[1 + e^{2i \operatorname{ArcTan}[ax]}]$$

Problem 48: Result more than twice size of optimal antiderivative.

$$\int \frac{e^{-2i \operatorname{ArcTan}[ax]}}{x} dx$$

Optimal (type 3, 14 leaves, 3 steps):

$$\operatorname{Log}[x] - 2 \operatorname{Log}[i - ax]$$

Result (type 3, 29 leaves):

$$\operatorname{Log}[1 - e^{-2i \operatorname{ArcTan}[ax]}] + \operatorname{Log}[1 + e^{-2i \operatorname{ArcTan}[ax]}]$$

Problem 61: Result is not expressed in closed-form.

$$\int e^{\frac{1}{2}i \operatorname{ArcTan}[ax]} x^2 dx$$

Optimal (type 3, 339 leaves, 15 steps):

$$\begin{aligned} & - \frac{3i(1-iax)^{3/4}(1+iax)^{1/4}}{8a^3} - \frac{i(1-iax)^{3/4}(1+iax)^{5/4}}{12a^3} + \\ & \frac{x(1-iax)^{3/4}(1+iax)^{5/4}}{3a^2} + \frac{3i \operatorname{ArcTan}\left[1 - \frac{\sqrt{2}(1-iax)^{1/4}}{(1+iax)^{1/4}}\right]}{8\sqrt{2}a^3} - \frac{3i \operatorname{ArcTan}\left[1 + \frac{\sqrt{2}(1-iax)^{1/4}}{(1+iax)^{1/4}}\right]}{8\sqrt{2}a^3} - \\ & \frac{3i \operatorname{Log}\left[1 + \frac{\sqrt{1-iax}}{\sqrt{1+iax}} - \frac{\sqrt{2}(1-iax)^{1/4}}{(1+iax)^{1/4}}\right]}{16\sqrt{2}a^3} + \frac{3i \operatorname{Log}\left[1 + \frac{\sqrt{1-iax}}{\sqrt{1+iax}} + \frac{\sqrt{2}(1-iax)^{1/4}}{(1+iax)^{1/4}}\right]}{16\sqrt{2}a^3} \end{aligned}$$

Result (type 7, 107 leaves):

$$\frac{1}{96 a^3} \left( -\frac{8 i e^{\frac{1}{2} i \text{ArcTan}[a x]} \left( 9 + 6 e^{2 i \text{ArcTan}[a x]} + 29 e^{4 i \text{ArcTan}[a x]} \right)}{\left( 1 + e^{2 i \text{ArcTan}[a x]} \right)^3} + \right. \\ \left. 9 \text{RootSum} \left[ 1 + \#1^4 \ \&, \frac{\text{ArcTan}[a x] + 2 i \text{Log} \left[ e^{\frac{1}{2} i \text{ArcTan}[a x]} - \#1 \right]}{\#1^3} \ \& \right] \right)$$

Problem 63: Result is not expressed in closed-form.

$$\int e^{\frac{1}{2} i \text{ArcTan}[a x]} dx$$

Optimal (type 3, 268 leaves, 13 steps):

$$\frac{i (1 - i a x)^{3/4} (1 + i a x)^{1/4}}{a} - \frac{i \text{ArcTan} \left[ 1 - \frac{\sqrt{2} (1 - i a x)^{1/4}}{(1 + i a x)^{1/4}} \right]}{\sqrt{2} a} + \frac{i \text{ArcTan} \left[ 1 + \frac{\sqrt{2} (1 - i a x)^{1/4}}{(1 + i a x)^{1/4}} \right]}{\sqrt{2} a} + \\ \frac{i \text{Log} \left[ 1 + \frac{\sqrt{1 - i a x}}{\sqrt{1 + i a x}} - \frac{\sqrt{2} (1 - i a x)^{1/4}}{(1 + i a x)^{1/4}} \right]}{2 \sqrt{2} a} - \frac{i \text{Log} \left[ 1 + \frac{\sqrt{1 - i a x}}{\sqrt{1 + i a x}} + \frac{\sqrt{2} (1 - i a x)^{1/4}}{(1 + i a x)^{1/4}} \right]}{2 \sqrt{2} a}$$

Result (type 7, 79 leaves):

$$-\frac{1}{4 a} \left( -\frac{8 i e^{\frac{1}{2} i \text{ArcTan}[a x]} \left( 1 + e^{2 i \text{ArcTan}[a x]} \right)}{1 + e^{2 i \text{ArcTan}[a x]}} + \text{RootSum} \left[ 1 + \#1^4 \ \&, \frac{\text{ArcTan}[a x] + 2 i \text{Log} \left[ e^{\frac{1}{2} i \text{ArcTan}[a x]} - \#1 \right]}{\#1^3} \ \& \right] \right)$$

Problem 71: Result is not expressed in closed-form.

$$\int e^{\frac{3}{2} i \text{ArcTan}[a x]} x^2 dx$$

Optimal (type 3, 339 leaves, 15 steps):

$$-\frac{17 i (1 - i a x)^{1/4} (1 + i a x)^{3/4}}{24 a^3} - \frac{i (1 - i a x)^{1/4} (1 + i a x)^{7/4}}{4 a^3} + \\ \frac{x (1 - i a x)^{1/4} (1 + i a x)^{7/4}}{3 a^2} + \frac{17 i \text{ArcTan} \left[ 1 - \frac{\sqrt{2} (1 - i a x)^{1/4}}{(1 + i a x)^{1/4}} \right]}{8 \sqrt{2} a^3} - \frac{17 i \text{ArcTan} \left[ 1 + \frac{\sqrt{2} (1 - i a x)^{1/4}}{(1 + i a x)^{1/4}} \right]}{8 \sqrt{2} a^3} + \\ \frac{17 i \text{Log} \left[ 1 + \frac{\sqrt{1 - i a x}}{\sqrt{1 + i a x}} - \frac{\sqrt{2} (1 - i a x)^{1/4}}{(1 + i a x)^{1/4}} \right]}{16 \sqrt{2} a^3} - \frac{17 i \text{Log} \left[ 1 + \frac{\sqrt{1 - i a x}}{\sqrt{1 + i a x}} + \frac{\sqrt{2} (1 - i a x)^{1/4}}{(1 + i a x)^{1/4}} \right]}{16 \sqrt{2} a^3}$$

Result (type 7, 107 leaves):

$$\frac{1}{96 a^3} \left( -\frac{8 i e^{\frac{3}{2} i \text{ArcTan}[a x]} (17 + 30 e^{2 i \text{ArcTan}[a x]} + 45 e^{4 i \text{ArcTan}[a x]})}{(1 + e^{2 i \text{ArcTan}[a x]})^3} + 51 \text{RootSum}\left[1 + \#1^4 \&, \frac{\text{ArcTan}[a x] + 2 i \text{Log}\left[e^{\frac{1}{2} i \text{ArcTan}[a x]} - \#1\right]}{\#1}\right] \& \right)$$

Problem 73: Result is not expressed in closed-form.

$$\int e^{\frac{3}{2} i \text{ArcTan}[a x]} dx$$

Optimal (type 3, 268 leaves, 13 steps):

$$\frac{i (1 - i a x)^{1/4} (1 + i a x)^{3/4}}{a} - \frac{3 i \text{ArcTan}\left[1 - \frac{\sqrt{2} (1 - i a x)^{1/4}}{(1 + i a x)^{1/4}}\right]}{\sqrt{2} a} + \frac{3 i \text{ArcTan}\left[1 + \frac{\sqrt{2} (1 - i a x)^{1/4}}{(1 + i a x)^{1/4}}\right]}{\sqrt{2} a} - \frac{3 i \text{Log}\left[1 + \frac{\sqrt{1 - i a x}}{\sqrt{1 + i a x}} - \frac{\sqrt{2} (1 - i a x)^{1/4}}{(1 + i a x)^{1/4}}\right]}{2 \sqrt{2} a} + \frac{3 i \text{Log}\left[1 + \frac{\sqrt{1 - i a x}}{\sqrt{1 + i a x}} + \frac{\sqrt{2} (1 - i a x)^{1/4}}{(1 + i a x)^{1/4}}\right]}{2 \sqrt{2} a}$$

Result (type 7, 82 leaves):

$$\frac{2 i e^{\frac{3}{2} i \text{ArcTan}[a x]}}{a (1 + e^{2 i \text{ArcTan}[a x]})} - \frac{3 \text{RootSum}\left[1 + \#1^4 \&, \frac{\text{ArcTan}[a x] + 2 i \text{Log}\left[e^{\frac{1}{2} i \text{ArcTan}[a x]} - \#1\right]}{\#1}\right] \&}{4 a}$$

Problem 80: Result is not expressed in closed-form.

$$\int e^{\frac{5}{2} i \text{ArcTan}[a x]} x^2 dx$$

Optimal (type 3, 371 leaves, 16 steps):

$$\frac{55 i (1 - i a x)^{3/4} (1 + i a x)^{1/4}}{8 a^3} + \frac{11 i (1 - i a x)^{3/4} (1 + i a x)^{5/4}}{4 a^3} + \frac{2 i (1 + i a x)^{9/4}}{a^3 (1 - i a x)^{1/4}} + \frac{i (1 - i a x)^{3/4} (1 + i a x)^{9/4}}{3 a^3} - \frac{55 i \text{ArcTan}\left[1 - \frac{\sqrt{2} (1 - i a x)^{1/4}}{(1 + i a x)^{1/4}}\right]}{8 \sqrt{2} a^3} + \frac{55 i \text{ArcTan}\left[1 + \frac{\sqrt{2} (1 - i a x)^{1/4}}{(1 + i a x)^{1/4}}\right]}{8 \sqrt{2} a^3} + \frac{55 i \text{Log}\left[1 + \frac{\sqrt{1 - i a x}}{\sqrt{1 + i a x}} - \frac{\sqrt{2} (1 - i a x)^{1/4}}{(1 + i a x)^{1/4}}\right]}{16 \sqrt{2} a^3} - \frac{55 i \text{Log}\left[1 + \frac{\sqrt{1 - i a x}}{\sqrt{1 + i a x}} + \frac{\sqrt{2} (1 - i a x)^{1/4}}{(1 + i a x)^{1/4}}\right]}{16 \sqrt{2} a^3}$$

Result (type 7, 120 leaves):

$$\frac{1}{a^3} \left( \left( i e^{\frac{1}{2} i \text{ArcTan}[a x]} \left( 165 + 462 e^{2 i \text{ArcTan}[a x]} + 425 e^{4 i \text{ArcTan}[a x]} + 96 e^{6 i \text{ArcTan}[a x]} \right) \right) / \right. \\ \left. \left( 12 \left( 1 + e^{2 i \text{ArcTan}[a x]} \right)^3 \right) - \frac{55}{32} \text{RootSum} \left[ 1 + \#1^4 \&, \frac{\text{ArcTan}[a x] + 2 i \text{Log} \left[ e^{\frac{1}{2} i \text{ArcTan}[a x]} - \#1 \right]}{\#1^3} \& \right] \right)$$

**Problem 82: Result is not expressed in closed-form.**

$$\int e^{\frac{5}{2} i \text{ArcTan}[a x]} dx$$

Optimal (type 3, 299 leaves, 14 steps):

$$-\frac{5 i (1 - i a x)^{3/4} (1 + i a x)^{1/4}}{a} - \frac{4 i (1 + i a x)^{5/4}}{a (1 - i a x)^{1/4}} + \\ \frac{5 i \text{ArcTan} \left[ 1 - \frac{\sqrt{2} (1 - i a x)^{1/4}}{(1 + i a x)^{1/4}} \right]}{\sqrt{2} a} - \frac{5 i \text{ArcTan} \left[ 1 + \frac{\sqrt{2} (1 - i a x)^{1/4}}{(1 + i a x)^{1/4}} \right]}{\sqrt{2} a} - \\ \frac{5 i \text{Log} \left[ 1 + \frac{\sqrt{1 - i a x}}{\sqrt{1 + i a x}} - \frac{\sqrt{2} (1 - i a x)^{1/4}}{(1 + i a x)^{1/4}} \right]}{2 \sqrt{2} a} + \frac{5 i \text{Log} \left[ 1 + \frac{\sqrt{1 - i a x}}{\sqrt{1 + i a x}} + \frac{\sqrt{2} (1 - i a x)^{1/4}}{(1 + i a x)^{1/4}} \right]}{2 \sqrt{2} a}$$

Result (type 7, 95 leaves):

$$\frac{1}{4 a} \left( - \frac{8 i e^{\frac{1}{2} i \text{ArcTan}[a x]} (5 + 4 e^{2 i \text{ArcTan}[a x]})}{1 + e^{2 i \text{ArcTan}[a x]}} + \right. \\ \left. 5 \text{RootSum} \left[ 1 + \#1^4 \&, \frac{\text{ArcTan}[a x] + 2 i \text{Log} \left[ e^{\frac{1}{2} i \text{ArcTan}[a x]} - \#1 \right]}{\#1^3} \& \right] \right)$$

**Problem 89: Result is not expressed in closed-form.**

$$\int e^{-\frac{1}{2} i \text{ArcTan}[a x]} x^2 dx$$

Optimal (type 3, 339 leaves, 15 steps):

$$\frac{3 i (1 - i a x)^{1/4} (1 + i a x)^{3/4}}{8 a^3} + \frac{i (1 - i a x)^{5/4} (1 + i a x)^{3/4}}{12 a^3} + \\ \frac{x (1 - i a x)^{5/4} (1 + i a x)^{3/4}}{3 a^2} + \frac{3 i \text{ArcTan} \left[ 1 - \frac{\sqrt{2} (1 - i a x)^{1/4}}{(1 + i a x)^{1/4}} \right]}{8 \sqrt{2} a^3} - \frac{3 i \text{ArcTan} \left[ 1 + \frac{\sqrt{2} (1 - i a x)^{1/4}}{(1 + i a x)^{1/4}} \right]}{8 \sqrt{2} a^3} + \\ \frac{3 i \text{Log} \left[ 1 + \frac{\sqrt{1 - i a x}}{\sqrt{1 + i a x}} - \frac{\sqrt{2} (1 - i a x)^{1/4}}{(1 + i a x)^{1/4}} \right]}{16 \sqrt{2} a^3} - \frac{3 i \text{Log} \left[ 1 + \frac{\sqrt{1 - i a x}}{\sqrt{1 + i a x}} + \frac{\sqrt{2} (1 - i a x)^{1/4}}{(1 + i a x)^{1/4}} \right]}{16 \sqrt{2} a^3}$$

Result (type 7, 107 leaves):

$$\frac{1}{96 a^3} \left( \frac{8 i e^{\frac{3}{2} i \text{ArcTan}[a x]} (29 + 6 e^{2 i \text{ArcTan}[a x]} + 9 e^{4 i \text{ArcTan}[a x]})}{(1 + e^{2 i \text{ArcTan}[a x]})^3} + \right. \\ \left. 9 \text{RootSum}\left[1 + \#1^4 \&, \frac{\text{ArcTan}[a x] - 2 i \text{Log}\left[e^{-\frac{1}{2} i \text{ArcTan}[a x]} - \#1\right]}{\#1^3} \&\right] \right)$$

Problem 91: Result is not expressed in closed-form.

$$\int e^{-\frac{1}{2} i \text{ArcTan}[a x]} dx$$

Optimal (type 3, 268 leaves, 13 steps):

$$\frac{i (1 - i a x)^{1/4} (1 + i a x)^{3/4}}{a} - \frac{i \text{ArcTan}\left[1 - \frac{\sqrt{2} (1 - i a x)^{1/4}}{(1 + i a x)^{1/4}}\right]}{\sqrt{2} a} + \frac{i \text{ArcTan}\left[1 + \frac{\sqrt{2} (1 - i a x)^{1/4}}{(1 + i a x)^{1/4}}\right]}{\sqrt{2} a} - \\ \frac{i \text{Log}\left[1 + \frac{\sqrt{1 - i a x}}{\sqrt{1 + i a x}} - \frac{\sqrt{2} (1 - i a x)^{1/4}}{(1 + i a x)^{1/4}}\right]}{2 \sqrt{2} a} + \frac{i \text{Log}\left[1 + \frac{\sqrt{1 - i a x}}{\sqrt{1 + i a x}} + \frac{\sqrt{2} (1 - i a x)^{1/4}}{(1 + i a x)^{1/4}}\right]}{2 \sqrt{2} a}$$

Result (type 7, 81 leaves):

$$\frac{1}{4 a} \left( - \frac{8 i e^{\frac{3}{2} i \text{ArcTan}[a x]}}{1 + e^{2 i \text{ArcTan}[a x]}} + \text{RootSum}\left[1 + \#1^4 \&, \frac{-\text{ArcTan}[a x] + 2 i \text{Log}\left[e^{-\frac{1}{2} i \text{ArcTan}[a x]} - \#1\right]}{\#1^3} \&\right] \right)$$

Problem 98: Result is not expressed in closed-form.

$$\int e^{-\frac{3}{2} i \text{ArcTan}[a x]} x^2 dx$$

Optimal (type 3, 339 leaves, 15 steps):

$$\frac{17 i (1 - i a x)^{3/4} (1 + i a x)^{1/4}}{24 a^3} + \frac{i (1 - i a x)^{7/4} (1 + i a x)^{1/4}}{4 a^3} + \\ \frac{x (1 - i a x)^{7/4} (1 + i a x)^{1/4}}{3 a^2} + \frac{17 i \text{ArcTan}\left[1 - \frac{\sqrt{2} (1 - i a x)^{1/4}}{(1 + i a x)^{1/4}}\right]}{8 \sqrt{2} a^3} - \frac{17 i \text{ArcTan}\left[1 + \frac{\sqrt{2} (1 - i a x)^{1/4}}{(1 + i a x)^{1/4}}\right]}{8 \sqrt{2} a^3} - \\ \frac{17 i \text{Log}\left[1 + \frac{\sqrt{1 - i a x}}{\sqrt{1 + i a x}} - \frac{\sqrt{2} (1 - i a x)^{1/4}}{(1 + i a x)^{1/4}}\right]}{16 \sqrt{2} a^3} + \frac{17 i \text{Log}\left[1 + \frac{\sqrt{1 - i a x}}{\sqrt{1 + i a x}} + \frac{\sqrt{2} (1 - i a x)^{1/4}}{(1 + i a x)^{1/4}}\right]}{16 \sqrt{2} a^3}$$

Result (type 7, 107 leaves):

$$\frac{1}{96 a^3} \left( \frac{8 i e^{\frac{1}{2} i \text{ArcTan}[a x]} (45 + 30 e^{2 i \text{ArcTan}[a x]} + 17 e^{4 i \text{ArcTan}[a x]})}{(1 + e^{2 i \text{ArcTan}[a x]})^3} + 51 \text{RootSum}\left[1 + \#1^4 \&, \frac{\text{ArcTan}[a x] - 2 i \text{Log}\left[e^{-\frac{1}{2} i \text{ArcTan}[a x]} - \#1\right]}{\#1}\right] \& \right)$$

Problem 100: Result is not expressed in closed-form.

$$\int e^{-\frac{3}{2} i \text{ArcTan}[a x]} dx$$

Optimal (type 3, 268 leaves, 13 steps):

$$\begin{aligned} & - \frac{i (1 - i a x)^{3/4} (1 + i a x)^{1/4}}{a} - \frac{3 i \text{ArcTan}\left[1 - \frac{\sqrt{2} (1 - i a x)^{1/4}}{(1 + i a x)^{1/4}}\right]}{\sqrt{2} a} + \frac{3 i \text{ArcTan}\left[1 + \frac{\sqrt{2} (1 - i a x)^{1/4}}{(1 + i a x)^{1/4}}\right]}{\sqrt{2} a} + \\ & \frac{3 i \text{Log}\left[1 + \frac{\sqrt{1 - i a x}}{\sqrt{1 + i a x}} - \frac{\sqrt{2} (1 - i a x)^{1/4}}{(1 + i a x)^{1/4}}\right]}{2 \sqrt{2} a} - \frac{3 i \text{Log}\left[1 + \frac{\sqrt{1 - i a x}}{\sqrt{1 + i a x}} + \frac{\sqrt{2} (1 - i a x)^{1/4}}{(1 + i a x)^{1/4}}\right]}{2 \sqrt{2} a} \end{aligned}$$

Result (type 7, 82 leaves):

$$- \frac{2 i e^{-\frac{3}{2} i \text{ArcTan}[a x]}}{a (1 + e^{-2 i \text{ArcTan}[a x]})} - \frac{3 \text{RootSum}\left[1 + \#1^4 \&, \frac{\text{ArcTan}[a x] - 2 i \text{Log}\left[e^{-\frac{1}{2} i \text{ArcTan}[a x]} - \#1\right]}{\#1}\right] \&}{4 a}$$

Problem 107: Result is not expressed in closed-form.

$$\int e^{-\frac{5}{2} i \text{ArcTan}[a x]} x^2 dx$$

Optimal (type 3, 371 leaves, 16 steps):

$$\begin{aligned} & - \frac{2 i (1 - i a x)^{9/4}}{a^3 (1 + i a x)^{1/4}} - \frac{55 i (1 - i a x)^{1/4} (1 + i a x)^{3/4}}{8 a^3} - \frac{11 i (1 - i a x)^{5/4} (1 + i a x)^{3/4}}{4 a^3} - \\ & \frac{i (1 - i a x)^{9/4} (1 + i a x)^{3/4}}{3 a^3} - \frac{55 i \text{ArcTan}\left[1 - \frac{\sqrt{2} (1 - i a x)^{1/4}}{(1 + i a x)^{1/4}}\right]}{8 \sqrt{2} a^3} + \frac{55 i \text{ArcTan}\left[1 + \frac{\sqrt{2} (1 - i a x)^{1/4}}{(1 + i a x)^{1/4}}\right]}{8 \sqrt{2} a^3} - \\ & \frac{55 i \text{Log}\left[1 + \frac{\sqrt{1 - i a x}}{\sqrt{1 + i a x}} - \frac{\sqrt{2} (1 - i a x)^{1/4}}{(1 + i a x)^{1/4}}\right]}{16 \sqrt{2} a^3} + \frac{55 i \text{Log}\left[1 + \frac{\sqrt{1 - i a x}}{\sqrt{1 + i a x}} + \frac{\sqrt{2} (1 - i a x)^{1/4}}{(1 + i a x)^{1/4}}\right]}{16 \sqrt{2} a^3} \end{aligned}$$

Result (type 7, 120 leaves):

$$\frac{1}{a^3} \left( - \left( \left( i e^{-\frac{1}{2} i \text{ArcTan}[a x]} \left( 96 + 425 e^{2 i \text{ArcTan}[a x]} + 462 e^{4 i \text{ArcTan}[a x]} + 165 e^{6 i \text{ArcTan}[a x]} \right) \right) / \right. \right. \\ \left. \left. \left( 12 \left( 1 + e^{2 i \text{ArcTan}[a x]} \right)^3 \right) \right) - \right. \\ \left. \frac{55}{32} \text{RootSum} \left[ 1 + \#1^4 \ \&, \frac{\text{ArcTan}[a x] - 2 i \text{Log} \left[ e^{-\frac{1}{2} i \text{ArcTan}[a x]} - \#1 \right]}{\#1^3} \ \& \right] \right)$$

**Problem 109: Result is not expressed in closed-form.**

$$\int e^{-\frac{5}{2} i \text{ArcTan}[a x]} dx$$

Optimal (type 3, 299 leaves, 14 steps):

$$\frac{4 i (1 - i a x)^{5/4}}{a (1 + i a x)^{1/4}} + \frac{5 i (1 - i a x)^{1/4} (1 + i a x)^{3/4}}{a} + \\ \frac{5 i \text{ArcTan} \left[ 1 - \frac{\sqrt{2} (1 - i a x)^{1/4}}{(1 + i a x)^{1/4}} \right]}{\sqrt{2} a} - \frac{5 i \text{ArcTan} \left[ 1 + \frac{\sqrt{2} (1 - i a x)^{1/4}}{(1 + i a x)^{1/4}} \right]}{\sqrt{2} a} + \\ \frac{5 i \text{Log} \left[ 1 + \frac{\sqrt{1 - i a x}}{\sqrt{1 + i a x}} - \frac{\sqrt{2} (1 - i a x)^{1/4}}{(1 + i a x)^{1/4}} \right]}{2 \sqrt{2} a} - \frac{5 i \text{Log} \left[ 1 + \frac{\sqrt{1 - i a x}}{\sqrt{1 + i a x}} + \frac{\sqrt{2} (1 - i a x)^{1/4}}{(1 + i a x)^{1/4}} \right]}{2 \sqrt{2} a}$$

Result (type 7, 95 leaves):

$$\frac{1}{4 a} \left( \frac{8 i e^{-\frac{1}{2} i \text{ArcTan}[a x]} (4 + 5 e^{2 i \text{ArcTan}[a x]})}{1 + e^{2 i \text{ArcTan}[a x]}} + \right. \\ \left. 5 \text{RootSum} \left[ 1 + \#1^4 \ \&, \frac{\text{ArcTan}[a x] - 2 i \text{Log} \left[ e^{-\frac{1}{2} i \text{ArcTan}[a x]} - \#1 \right]}{\#1^3} \ \& \right] \right)$$

**Problem 115: Result is not expressed in closed-form.**

$$\int e^{\frac{1}{3} i \text{ArcTan}[x]} x^2 dx$$

Optimal (type 3, 319 leaves, 16 steps):

$$\begin{aligned}
 & -\frac{19}{54} i (1-i x)^{5/6} (1+i x)^{1/6} - \frac{1}{18} i (1-i x)^{5/6} (1+i x)^{7/6} + \\
 & \frac{1}{3} (1-i x)^{5/6} (1+i x)^{7/6} x + \frac{19}{162} i \operatorname{ArcTan}\left[\sqrt{3} - \frac{2(1-i x)^{1/6}}{(1+i x)^{1/6}}\right] - \\
 & \frac{19}{162} i \operatorname{ArcTan}\left[\sqrt{3} + \frac{2(1-i x)^{1/6}}{(1+i x)^{1/6}}\right] - \frac{19}{81} i \operatorname{ArcTan}\left[\frac{(1-i x)^{1/6}}{(1+i x)^{1/6}}\right] - \\
 & \frac{19 i \operatorname{Log}\left[1 + \frac{(1-i x)^{1/3}}{(1+i x)^{1/3}} - \frac{\sqrt{3}(1-i x)^{1/6}}{(1+i x)^{1/6}}\right]}{108 \sqrt{3}} + \frac{19 i \operatorname{Log}\left[1 + \frac{(1-i x)^{1/3}}{(1+i x)^{1/3}} + \frac{\sqrt{3}(1-i x)^{1/6}}{(1+i x)^{1/6}}\right]}{108 \sqrt{3}}
 \end{aligned}$$

Result (type 7, 156 leaves):

$$\begin{aligned}
 & \frac{1}{486} \left( -6 i \left( \frac{3 e^{\frac{1}{3} i \operatorname{ArcTan}[x]} (19 + 8 e^{2 i \operatorname{ArcTan}[x]} + 61 e^{4 i \operatorname{ArcTan}[x]})}{(1 + e^{2 i \operatorname{ArcTan}[x]})^3} - 19 \operatorname{ArcTan}\left[e^{\frac{1}{3} i \operatorname{ArcTan}[x]}\right] \right) - \right. \\
 & \left. 19 \operatorname{RootSum}\left[1 - \#1^2 + \#1^4 \&, \frac{1}{-\#1 + 2 \#1^3} \left( -2 \operatorname{ArcTan}[x] - \right. \right. \right. \\
 & \left. \left. \left. 6 i \operatorname{Log}\left[e^{\frac{1}{3} i \operatorname{ArcTan}[x]} - \#1\right] + \operatorname{ArcTan}[x] \#1^2 + 3 i \operatorname{Log}\left[e^{\frac{1}{3} i \operatorname{ArcTan}[x]} - \#1\right] \#1^2\right) \& \right] \right)
 \end{aligned}$$

Problem 117: Result is not expressed in closed-form.

$$\int e^{\frac{1}{3} i \operatorname{ArcTan}[x]} dx$$

Optimal (type 3, 262 leaves, 14 steps):

$$\begin{aligned}
 & i (1-i x)^{5/6} (1+i x)^{1/6} - \frac{1}{3} i \operatorname{ArcTan}\left[\sqrt{3} - \frac{2(1-i x)^{1/6}}{(1+i x)^{1/6}}\right] + \frac{1}{3} i \operatorname{ArcTan}\left[\sqrt{3} + \frac{2(1-i x)^{1/6}}{(1+i x)^{1/6}}\right] + \\
 & \frac{2}{3} i \operatorname{ArcTan}\left[\frac{(1-i x)^{1/6}}{(1+i x)^{1/6}}\right] + \frac{i \operatorname{Log}\left[1 + \frac{(1-i x)^{1/3}}{(1+i x)^{1/3}} - \frac{\sqrt{3}(1-i x)^{1/6}}{(1+i x)^{1/6}}\right]}{2 \sqrt{3}} - \frac{i \operatorname{Log}\left[1 + \frac{(1-i x)^{1/3}}{(1+i x)^{1/3}} + \frac{\sqrt{3}(1-i x)^{1/6}}{(1+i x)^{1/6}}\right]}{2 \sqrt{3}}
 \end{aligned}$$

Result (type 7, 133 leaves):

$$\begin{aligned}
 & \frac{2 i e^{\frac{1}{3} i \operatorname{ArcTan}[x]}}{1 + e^{2 i \operatorname{ArcTan}[x]}} - \frac{2}{3} i \operatorname{ArcTan}\left[e^{\frac{1}{3} i \operatorname{ArcTan}[x]}\right] + \frac{1}{9} \operatorname{RootSum}\left[1 - \#1^2 + \#1^4 \&, \frac{1}{-\#1 + 2 \#1^3} \right. \\
 & \left. \left( -2 \operatorname{ArcTan}[x] - 6 i \operatorname{Log}\left[e^{\frac{1}{3} i \operatorname{ArcTan}[x]} - \#1\right] + \operatorname{ArcTan}[x] \#1^2 + 3 i \operatorname{Log}\left[e^{\frac{1}{3} i \operatorname{ArcTan}[x]} - \#1\right] \#1^2\right) \& \right]
 \end{aligned}$$

Problem 122: Result is not expressed in closed-form.

$$\int e^{\frac{2}{3} i \operatorname{ArcTan}[x]} x^2 dx$$

Optimal (type 3, 177 leaves, 5 steps):



$$\begin{aligned}
 & -\frac{11}{27} i (1-i x)^{2/3} (1+i x)^{1/3} - \frac{1}{9} i (1-i x)^{2/3} (1+i x)^{4/3} + \frac{1}{3} (1-i x)^{2/3} (1+i x)^{4/3} x + \\
 & \frac{22 i \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} - \frac{2(1-i x)^{1/3}}{\sqrt{3}(1+i x)^{1/3}}\right]}{27 \sqrt{3}} + \frac{11}{27} i \operatorname{Log}\left[1 + \frac{(1-i x)^{1/3}}{(1+i x)^{1/3}}\right] + \frac{11}{81} i \operatorname{Log}[1+i x]
 \end{aligned}$$

Result (type 7, 154 leaves):

$$\begin{aligned}
 & \frac{2}{243} \left( -\frac{9 i e^{\frac{2}{3} i \operatorname{ArcTan}[x]} (11 + 10 e^{2 i \operatorname{ArcTan}[x]} + 35 e^{4 i \operatorname{ArcTan}[x]})}{(1 + e^{2 i \operatorname{ArcTan}[x]})^3} + \right. \\
 & \left. 22 \operatorname{ArcTan}[x] + 33 i \operatorname{Log}\left[1 + e^{\frac{2}{3} i \operatorname{ArcTan}[x]}\right] + 11 \operatorname{RootSum}\left[1 - \#1^2 + \#1^4 \&, \frac{1}{-2 + \#1^2}\right. \right. \\
 & \left. \left. \left( \operatorname{ArcTan}[x] + 3 i \operatorname{Log}\left[e^{\frac{1}{3} i \operatorname{ArcTan}[x]} - \#1\right] + \operatorname{ArcTan}[x] \#1^2 + 3 i \operatorname{Log}\left[e^{\frac{1}{3} i \operatorname{ArcTan}[x]} - \#1\right] \#1^2\right) \& \right] \right)
 \end{aligned}$$

**Problem 124: Result is not expressed in closed-form.**

$$\int e^{\frac{2}{3} i \operatorname{ArcTan}[x]} dx$$

Optimal (type 3, 116 leaves, 3 steps):

$$i (1-i x)^{2/3} (1+i x)^{1/3} - \frac{2 i \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} - \frac{2(1-i x)^{1/3}}{\sqrt{3}(1+i x)^{1/3}}\right]}{\sqrt{3}} - i \operatorname{Log}\left[1 + \frac{(1-i x)^{1/3}}{(1+i x)^{1/3}}\right] - \frac{1}{3} i \operatorname{Log}[1+i x]$$

Result (type 7, 134 leaves):

$$\begin{aligned}
 & \frac{2 i e^{\frac{2}{3} i \operatorname{ArcTan}[x]}}{1 + e^{2 i \operatorname{ArcTan}[x]}} - \frac{4 \operatorname{ArcTan}[x]}{9} - \frac{2}{3} i \operatorname{Log}\left[1 + e^{\frac{2}{3} i \operatorname{ArcTan}[x]}\right] - \frac{2}{9} \operatorname{RootSum}\left[1 - \#1^2 + \#1^4 \&, \frac{1}{-2 + \#1^2}\right. \\
 & \left. \left( \operatorname{ArcTan}[x] + 3 i \operatorname{Log}\left[e^{\frac{1}{3} i \operatorname{ArcTan}[x]} - \#1\right] + \operatorname{ArcTan}[x] \#1^2 + 3 i \operatorname{Log}\left[e^{\frac{1}{3} i \operatorname{ArcTan}[x]} - \#1\right] \#1^2\right) \& \right]
 \end{aligned}$$

**Problem 128: Result is not expressed in closed-form.**

$$\int e^{\frac{1}{4} i \operatorname{ArcTan}[a x]} x^2 dx$$

Optimal (type 3, 741 leaves, 27 steps):

$$\begin{aligned}
 & - \frac{11 i (1 - i a x)^{7/8} (1 + i a x)^{1/8}}{32 a^3} - \frac{i (1 - i a x)^{7/8} (1 + i a x)^{9/8}}{24 a^3} + \\
 & \frac{x (1 - i a x)^{7/8} (1 + i a x)^{9/8}}{3 a^2} + \frac{11 i \sqrt{2 + \sqrt{2}} \operatorname{ArcTan}\left[\frac{\sqrt{2 - \sqrt{2}} - 2 (1 - i a x)^{1/8}}{(1 + i a x)^{1/8}}\right]}{128 a^3} + \\
 & \frac{11 i \sqrt{2 - \sqrt{2}} \operatorname{ArcTan}\left[\frac{\sqrt{2 + \sqrt{2}} - 2 (1 - i a x)^{1/8}}{\sqrt{2 - \sqrt{2}}}\right]}{128 a^3} - \frac{11 i \sqrt{2 + \sqrt{2}} \operatorname{ArcTan}\left[\frac{\sqrt{2 - \sqrt{2}} + 2 (1 - i a x)^{1/8}}{\sqrt{2 + \sqrt{2}}}\right]}{128 a^3} - \\
 & \frac{11 i \sqrt{2 - \sqrt{2}} \operatorname{ArcTan}\left[\frac{\sqrt{2 + \sqrt{2}} + 2 (1 - i a x)^{1/8}}{\sqrt{2 - \sqrt{2}}}\right]}{128 a^3} - \frac{11 i \sqrt{2 - \sqrt{2}} \operatorname{Log}\left[1 + \frac{(1 - i a x)^{1/4}}{(1 + i a x)^{1/4}} - \frac{\sqrt{2 - \sqrt{2}} (1 - i a x)^{1/8}}{(1 + i a x)^{1/8}}\right]}{256 a^3} + \\
 & \frac{11 i \sqrt{2 - \sqrt{2}} \operatorname{Log}\left[1 + \frac{(1 - i a x)^{1/4}}{(1 + i a x)^{1/4}} + \frac{\sqrt{2 - \sqrt{2}} (1 - i a x)^{1/8}}{(1 + i a x)^{1/8}}\right]}{256 a^3} - \\
 & \frac{11 i \sqrt{2 + \sqrt{2}} \operatorname{Log}\left[1 + \frac{(1 - i a x)^{1/4}}{(1 + i a x)^{1/4}} - \frac{\sqrt{2 + \sqrt{2}} (1 - i a x)^{1/8}}{(1 + i a x)^{1/8}}\right]}{256 a^3} + \\
 & \frac{11 i \sqrt{2 + \sqrt{2}} \operatorname{Log}\left[1 + \frac{(1 - i a x)^{1/4}}{(1 + i a x)^{1/4}} + \frac{\sqrt{2 + \sqrt{2}} (1 - i a x)^{1/8}}{(1 + i a x)^{1/8}}\right]}{256 a^3}
 \end{aligned}$$

Result (type 7, 108 leaves):

$$\begin{aligned}
 & \frac{1}{a^3} \left( - \frac{i e^{\frac{1}{4} i \operatorname{ArcTan}[a x]} (33 + 10 e^{2 i \operatorname{ArcTan}[a x]} + 105 e^{4 i \operatorname{ArcTan}[a x]})}{48 (1 + e^{2 i \operatorname{ArcTan}[a x]})^3} + \right. \\
 & \left. \frac{11}{512} \operatorname{RootSum}\left[1 + \#1^8 \&, \frac{\operatorname{ArcTan}[a x] + 4 i \operatorname{Log}\left[e^{\frac{1}{4} i \operatorname{ArcTan}[a x]} - \#1\right]}{\#1^7} \&\right] \right)
 \end{aligned}$$

Problem 129: Result is not expressed in closed-form.

$$\int e^{\frac{1}{4} i \operatorname{ArcTan}[a x]} x \, dx$$

Optimal (type 3, 689 leaves, 26 steps):

$$\begin{aligned} & \frac{(1 - i a x)^{7/8} (1 + i a x)^{1/8}}{8 a^2} + \frac{(1 - i a x)^{7/8} (1 + i a x)^{9/8}}{2 a^2} - \\ & \frac{\sqrt{2 + \sqrt{2}} \operatorname{ArcTan}\left[\frac{\sqrt{2 - \sqrt{2}} - 2(1 - i a x)^{1/8}}{(1 + i a x)^{1/8}}\right]}{32 a^2} - \frac{\sqrt{2 - \sqrt{2}} \operatorname{ArcTan}\left[\frac{\sqrt{2 + \sqrt{2}} - 2(1 - i a x)^{1/8}}{\sqrt{2 - \sqrt{2}}}\right]}{32 a^2} + \\ & \frac{\sqrt{2 + \sqrt{2}} \operatorname{ArcTan}\left[\frac{\sqrt{2 - \sqrt{2}} + 2(1 - i a x)^{1/8}}{(1 + i a x)^{1/8}}\right]}{32 a^2} + \frac{\sqrt{2 - \sqrt{2}} \operatorname{ArcTan}\left[\frac{\sqrt{2 + \sqrt{2}} + 2(1 - i a x)^{1/8}}{\sqrt{2 - \sqrt{2}}}\right]}{32 a^2} + \\ & \frac{\sqrt{2 - \sqrt{2}} \operatorname{Log}\left[1 + \frac{(1 - i a x)^{1/4}}{(1 + i a x)^{1/4}} - \frac{\sqrt{2 - \sqrt{2}} (1 - i a x)^{1/8}}{(1 + i a x)^{1/8}}\right]}{64 a^2} - \frac{\sqrt{2 - \sqrt{2}} \operatorname{Log}\left[1 + \frac{(1 - i a x)^{1/4}}{(1 + i a x)^{1/4}} + \frac{\sqrt{2 - \sqrt{2}} (1 - i a x)^{1/8}}{(1 + i a x)^{1/8}}\right]}{64 a^2} + \\ & \frac{\sqrt{2 + \sqrt{2}} \operatorname{Log}\left[1 + \frac{(1 - i a x)^{1/4}}{(1 + i a x)^{1/4}} - \frac{\sqrt{2 + \sqrt{2}} (1 - i a x)^{1/8}}{(1 + i a x)^{1/8}}\right]}{64 a^2} - \frac{\sqrt{2 + \sqrt{2}} \operatorname{Log}\left[1 + \frac{(1 - i a x)^{1/4}}{(1 + i a x)^{1/4}} + \frac{\sqrt{2 + \sqrt{2}} (1 - i a x)^{1/8}}{(1 + i a x)^{1/8}}\right]}{64 a^2} \end{aligned}$$

Result (type 7, 138 leaves):

$$\begin{aligned} & \frac{1}{128 a^2} \left( \frac{32 e^{\frac{1}{4} i \operatorname{ArcTan}[a x]} (1 + 9 e^{2 i \operatorname{ArcTan}[a x]})}{(1 + e^{2 i \operatorname{ArcTan}[a x]})^2} + \right. \\ & \operatorname{RootSum}\left[-i + \#1^4 \&, \frac{\operatorname{ArcTan}[a x] + 4 i \operatorname{Log}\left[e^{\frac{1}{4} i \operatorname{ArcTan}[a x]} - \#1\right]}{\#1^3} \& \right] - \\ & \left. \operatorname{RootSum}\left[i + \#1^4 \&, \frac{\operatorname{ArcTan}[a x] + 4 i \operatorname{Log}\left[e^{\frac{1}{4} i \operatorname{ArcTan}[a x]} - \#1\right]}{\#1^3} \& \right] \right) \end{aligned}$$

Problem 130: Result is not expressed in closed-form.

$$\int e^{\frac{1}{4} i \operatorname{ArcTan}[a x]} dx$$

Optimal (type 3, 674 leaves, 25 steps):

$$\begin{aligned}
 & \frac{i (1 - i a x)^{7/8} (1 + i a x)^{1/8}}{a} - \frac{i \sqrt{2 + \sqrt{2}} \operatorname{ArcTan}\left[\frac{\sqrt{2 - \sqrt{2}} - 2 (1 - i a x)^{1/8}}{(1 + i a x)^{1/8}}\right]}{4 a} - \\
 & \frac{i \sqrt{2 - \sqrt{2}} \operatorname{ArcTan}\left[\frac{\sqrt{2 + \sqrt{2}} - 2 (1 - i a x)^{1/8}}{\sqrt{2 - \sqrt{2}}}\right]}{4 a} + \frac{i \sqrt{2 + \sqrt{2}} \operatorname{ArcTan}\left[\frac{\sqrt{2 - \sqrt{2}} + 2 (1 - i a x)^{1/8}}{\sqrt{2 + \sqrt{2}}}\right]}{4 a} + \\
 & \frac{i \sqrt{2 - \sqrt{2}} \operatorname{ArcTan}\left[\frac{\sqrt{2 + \sqrt{2}} + 2 (1 - i a x)^{1/8}}{\sqrt{2 - \sqrt{2}}}\right]}{4 a} + \frac{i \sqrt{2 - \sqrt{2}} \operatorname{Log}\left[1 + \frac{(1 - i a x)^{1/4}}{(1 + i a x)^{1/4}} - \frac{\sqrt{2 - \sqrt{2}} (1 - i a x)^{1/8}}{(1 + i a x)^{1/8}}\right]}{8 a} - \\
 & \frac{i \sqrt{2 - \sqrt{2}} \operatorname{Log}\left[1 + \frac{(1 - i a x)^{1/4}}{(1 + i a x)^{1/4}} + \frac{\sqrt{2 - \sqrt{2}} (1 - i a x)^{1/8}}{(1 + i a x)^{1/8}}\right]}{8 a} + \\
 & \frac{i \sqrt{2 + \sqrt{2}} \operatorname{Log}\left[1 + \frac{(1 - i a x)^{1/4}}{(1 + i a x)^{1/4}} - \frac{\sqrt{2 + \sqrt{2}} (1 - i a x)^{1/8}}{(1 + i a x)^{1/8}}\right]}{8 a} - \\
 & \frac{i \sqrt{2 + \sqrt{2}} \operatorname{Log}\left[1 + \frac{(1 - i a x)^{1/4}}{(1 + i a x)^{1/4}} + \frac{\sqrt{2 + \sqrt{2}} (1 - i a x)^{1/8}}{(1 + i a x)^{1/8}}\right]}{8 a}
 \end{aligned}$$

Result (type 7, 79 leaves):

$$-\frac{1}{16 a} \left( -\frac{32 i e^{\frac{1}{4} i \operatorname{ArcTan}[a x]}}{1 + e^{2 i \operatorname{ArcTan}[a x]}} + \operatorname{RootSum}\left[1 + \#1^8 \&, \frac{\operatorname{ArcTan}[a x] + 4 i \operatorname{Log}\left[e^{\frac{1}{4} i \operatorname{ArcTan}[a x]} - \#1\right]}{\#1^7} \&\right] \right)$$

Problem 131: Result is not expressed in closed-form.

$$\int \frac{e^{\frac{1}{4} i \operatorname{ArcTan}[a x]}}{x} dx$$

Optimal (type 3, 859 leaves, 39 steps):

$$\begin{aligned}
 & -2 \operatorname{ArcTan}\left[\frac{(1+i a x)^{1/8}}{(1-i a x)^{1/8}}\right] + \sqrt{2+\sqrt{2}} \operatorname{ArcTan}\left[\frac{\sqrt{2-\sqrt{2}} - \frac{2(1-i a x)^{1/8}}{(1+i a x)^{1/8}}}{\sqrt{2+\sqrt{2}}}\right] + \\
 & \sqrt{2-\sqrt{2}} \operatorname{ArcTan}\left[\frac{\sqrt{2+\sqrt{2}} - \frac{2(1-i a x)^{1/8}}{(1+i a x)^{1/8}}}{\sqrt{2-\sqrt{2}}}\right] - \sqrt{2+\sqrt{2}} \operatorname{ArcTan}\left[\frac{\sqrt{2-\sqrt{2}} + \frac{2(1-i a x)^{1/8}}{(1+i a x)^{1/8}}}{\sqrt{2+\sqrt{2}}}\right] - \\
 & \sqrt{2-\sqrt{2}} \operatorname{ArcTan}\left[\frac{\sqrt{2+\sqrt{2}} + \frac{2(1-i a x)^{1/8}}{(1+i a x)^{1/8}}}{\sqrt{2-\sqrt{2}}}\right] + \sqrt{2} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2}(1+i a x)^{1/8}}{(1-i a x)^{1/8}}\right] - \\
 & \sqrt{2} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2}(1+i a x)^{1/8}}{(1-i a x)^{1/8}}\right] - 2 \operatorname{ArcTanh}\left[\frac{(1+i a x)^{1/8}}{(1-i a x)^{1/8}}\right] - \\
 & \frac{1}{2} \sqrt{2-\sqrt{2}} \operatorname{Log}\left[1 + \frac{(1-i a x)^{1/4}}{(1+i a x)^{1/4}} - \frac{\sqrt{2-\sqrt{2}}(1-i a x)^{1/8}}{(1+i a x)^{1/8}}\right] + \\
 & \frac{1}{2} \sqrt{2-\sqrt{2}} \operatorname{Log}\left[1 + \frac{(1-i a x)^{1/4}}{(1+i a x)^{1/4}} + \frac{\sqrt{2-\sqrt{2}}(1-i a x)^{1/8}}{(1+i a x)^{1/8}}\right] - \\
 & \frac{1}{2} \sqrt{2+\sqrt{2}} \operatorname{Log}\left[1 + \frac{(1-i a x)^{1/4}}{(1+i a x)^{1/4}} - \frac{\sqrt{2+\sqrt{2}}(1-i a x)^{1/8}}{(1+i a x)^{1/8}}\right] + \\
 & \frac{1}{2} \sqrt{2+\sqrt{2}} \operatorname{Log}\left[1 + \frac{(1-i a x)^{1/4}}{(1+i a x)^{1/4}} + \frac{\sqrt{2+\sqrt{2}}(1-i a x)^{1/8}}{(1+i a x)^{1/8}}\right] + \\
 & \frac{\operatorname{Log}\left[1 - \frac{\sqrt{2}(1+i a x)^{1/8}}{(1-i a x)^{1/8}} + \frac{(1+i a x)^{1/4}}{(1-i a x)^{1/4}}\right]}{\sqrt{2}} - \frac{\operatorname{Log}\left[1 + \frac{\sqrt{2}(1+i a x)^{1/8}}{(1-i a x)^{1/8}} + \frac{(1+i a x)^{1/4}}{(1-i a x)^{1/4}}\right]}{\sqrt{2}}
 \end{aligned}$$

Result (type 7, 252 leaves):

$$\begin{aligned}
 & -2 \operatorname{ArcTan}\left[e^{\frac{1}{4} i \operatorname{ArcTan}[a x]}\right] + \operatorname{Log}\left[1 - e^{\frac{1}{4} i \operatorname{ArcTan}[a x]}\right] + (-1)^{1/4} \operatorname{Log}\left[(-1)^{1/4} - e^{\frac{1}{4} i \operatorname{ArcTan}[a x]}\right] + \\
 & (-1)^{3/4} \operatorname{Log}\left[(-1)^{3/4} - e^{\frac{1}{4} i \operatorname{ArcTan}[a x]}\right] - \operatorname{Log}\left[1 + e^{\frac{1}{4} i \operatorname{ArcTan}[a x]}\right] - \\
 & (-1)^{1/4} \operatorname{Log}\left[(-1)^{1/4} + e^{\frac{1}{4} i \operatorname{ArcTan}[a x]}\right] - (-1)^{3/4} \operatorname{Log}\left[(-1)^{3/4} + e^{\frac{1}{4} i \operatorname{ArcTan}[a x]}\right] + \\
 & \frac{1}{4} \operatorname{RootSum}\left[-i + \#1^4 \&, \frac{-\operatorname{ArcTan}[a x] - 4 i \operatorname{Log}\left[e^{\frac{1}{4} i \operatorname{ArcTan}[a x]} - \#1\right]}{\#1^3} \&\right] + \\
 & \frac{1}{4} \operatorname{RootSum}\left[i + \#1^4 \&, \frac{\operatorname{ArcTan}[a x] + 4 i \operatorname{Log}\left[e^{\frac{1}{4} i \operatorname{ArcTan}[a x]} - \#1\right]}{\#1^3} \&\right]
 \end{aligned}$$

Problem 132: Result is not expressed in closed-form.

$$\int \frac{e^{\frac{1}{4} i \text{ArcTan}[a x]}}{x^2} dx$$

Optimal (type 3, 328 leaves, 16 steps):

$$\begin{aligned} & -\frac{(1-i a x)^{7/8} (1+i a x)^{1/8}}{x} - \frac{1}{2} i a \text{ArcTan}\left[\frac{(1+i a x)^{1/8}}{(1-i a x)^{1/8}}\right] + \\ & \frac{i a \text{ArcTan}\left[1 - \frac{\sqrt{2} (1+i a x)^{1/8}}{(1-i a x)^{1/8}}\right]}{2 \sqrt{2}} - \frac{i a \text{ArcTan}\left[1 + \frac{\sqrt{2} (1+i a x)^{1/8}}{(1-i a x)^{1/8}}\right]}{2 \sqrt{2}} - \frac{1}{2} i a \text{ArcTanh}\left[\frac{(1+i a x)^{1/8}}{(1-i a x)^{1/8}}\right] + \\ & \frac{i a \text{Log}\left[1 - \frac{\sqrt{2} (1+i a x)^{1/8}}{(1-i a x)^{1/8}} + \frac{(1+i a x)^{1/4}}{(1-i a x)^{1/4}}\right]}{4 \sqrt{2}} - \frac{i a \text{Log}\left[1 + \frac{\sqrt{2} (1+i a x)^{1/8}}{(1-i a x)^{1/8}} + \frac{(1+i a x)^{1/4}}{(1-i a x)^{1/4}}\right]}{4 \sqrt{2}} \end{aligned}$$

Result (type 7, 131 leaves):

$$\begin{aligned} & \frac{1}{16} a \left( -4 i \right. \\ & \left. \left( \frac{8 e^{\frac{1}{4} i \text{ArcTan}[a x]}}{-1 + e^{2 i \text{ArcTan}[a x]}} + 2 \text{ArcTan}\left[e^{\frac{1}{4} i \text{ArcTan}[a x]}\right] - \text{Log}\left[1 - e^{\frac{1}{4} i \text{ArcTan}[a x]}\right] + \text{Log}\left[1 + e^{\frac{1}{4} i \text{ArcTan}[a x]}\right] \right) - \right. \\ & \left. \text{RootSum}\left[1 + \#1^4 \&, \frac{\text{ArcTan}[a x] + 4 i \text{Log}\left[e^{\frac{1}{4} i \text{ArcTan}[a x]} - \#1\right]}{\#1^3} \&\right] \right) \end{aligned}$$

Problem 140: Result unnecessarily involves higher level functions.

$$\int e^{3 i \text{ArcTan}[a x]} x^m dx$$

Optimal (type 5, 159 leaves, 9 steps):

$$\begin{aligned} & -\frac{3 x^{1+m} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, -a^2 x^2\right]}{1+m} - \frac{i a x^{2+m} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, -a^2 x^2\right]}{2+m} + \\ & \frac{4 x^{1+m} \text{Hypergeometric2F1}\left[\frac{3}{2}, \frac{1+m}{2}, \frac{3+m}{2}, -a^2 x^2\right]}{1+m} + \frac{4 i a x^{2+m} \text{Hypergeometric2F1}\left[\frac{3}{2}, \frac{2+m}{2}, \frac{4+m}{2}, -a^2 x^2\right]}{2+m} \end{aligned}$$

Result (type 6, 315 leaves):

$$\begin{aligned}
 & \left( 2 (2+m) x^{1+m} \sqrt{-i+a x} \right. \\
 & \left. \left( - \left( \left( 2 \operatorname{AppellF1} \left[ 1+m, -\frac{1}{2}, \frac{3}{2}, 2+m, -i a x, i a x \right] \right) / \left( 2 (2+m) \operatorname{AppellF1} \left[ 1+m, -\frac{1}{2}, \frac{3}{2}, \right. \right. \right. \right. \right. \\
 & \quad \left. \left. \left. 2+m, -i a x, i a x \right] + i a x \left( 3 \operatorname{AppellF1} \left[ 2+m, -\frac{1}{2}, \frac{5}{2}, 3+m, -i a x, i a x \right] + \right. \right. \right. \\
 & \quad \left. \left. \left. \operatorname{AppellF1} \left[ 2+m, \frac{1}{2}, \frac{3}{2}, 3+m, -i a x, i a x \right] \right) \right) \right) - \right. \\
 & \left. \left( i \sqrt{1-i a x} \sqrt{1+a^2 x^2} \operatorname{AppellF1} \left[ 1+m, -\frac{1}{2}, \frac{1}{2}, 2+m, -i a x, i a x \right] \right) / \right. \\
 & \left. \left( \sqrt{1+i a x} \left( -2 i (2+m) \operatorname{AppellF1} \left[ 1+m, -\frac{1}{2}, \frac{1}{2}, 2+m, -i a x, i a x \right] + \right. \right. \right. \\
 & \quad \left. \left. \left. a x \left( \operatorname{AppellF1} \left[ 2+m, -\frac{1}{2}, \frac{3}{2}, 3+m, -i a x, i a x \right] + \right. \right. \right. \right. \\
 & \quad \left. \left. \left. \operatorname{HypergeometricPFQ} \left[ \left\{ \frac{1}{2}, 1+\frac{m}{2} \right\}, \left\{ 2+\frac{m}{2} \right\}, -a^2 x^2 \right] \right) \right) \right) \right) / \left( (1+m) (i+a x)^{3/2} \right)
 \end{aligned}$$

**Problem 141: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int e^{i \operatorname{ArcTan}[a x]} x^m dx$$

Optimal (type 5, 79 leaves, 4 steps):

$$\frac{x^{1+m} \operatorname{Hypergeometric2F1} \left[ \frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, -a^2 x^2 \right]}{1+m} + \frac{i a x^{2+m} \operatorname{Hypergeometric2F1} \left[ \frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, -a^2 x^2 \right]}{2+m}$$

Result (type 6, 193 leaves):

$$\begin{aligned}
 & \left( 2 i (2+m) x^{1+m} \sqrt{1-i a x} \sqrt{-i+a x} \sqrt{1+a^2 x^2} \operatorname{AppellF1} \left[ 1+m, -\frac{1}{2}, \frac{1}{2}, 2+m, -i a x, i a x \right] \right) / \\
 & \left( (1+m) \sqrt{1+i a x} (i+a x)^{3/2} \right. \\
 & \left. \left( -2 i (2+m) \operatorname{AppellF1} \left[ 1+m, -\frac{1}{2}, \frac{1}{2}, 2+m, -i a x, i a x \right] + a x \left( \operatorname{AppellF1} \left[ 2+m, -\frac{1}{2}, \right. \right. \right. \right. \right. \\
 & \quad \left. \left. \left. \frac{3}{2}, 3+m, -i a x, i a x \right] + \operatorname{HypergeometricPFQ} \left[ \left\{ \frac{1}{2}, 1+\frac{m}{2} \right\}, \left\{ 2+\frac{m}{2} \right\}, -a^2 x^2 \right] \right) \right) \right)
 \end{aligned}$$

**Problem 142: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int e^{-i \operatorname{ArcTan}[a x]} x^m dx$$

Optimal (type 5, 79 leaves, 4 steps):

$$\frac{x^{1+m} \operatorname{Hypergeometric2F1} \left[ \frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, -a^2 x^2 \right]}{1+m} - \frac{i a x^{2+m} \operatorname{Hypergeometric2F1} \left[ \frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, -a^2 x^2 \right]}{2+m}$$

Result (type 6, 193 leaves):

$$\begin{aligned}
 & - \left( \left( 2 i (2+m) x^{1+m} \sqrt{1+i a x} \sqrt{i+a x} \sqrt{1+a^2 x^2} \operatorname{AppellF1} \left[ 1+m, \frac{1}{2}, -\frac{1}{2}, 2+m, -i a x, i a x \right] \right) / \right. \\
 & \quad \left( (1+m) \sqrt{1-i a x} (-i+a x)^{3/2} \right. \\
 & \quad \left. \left( 2 i (2+m) \operatorname{AppellF1} \left[ 1+m, \frac{1}{2}, -\frac{1}{2}, 2+m, -i a x, i a x \right] + a x \left( \operatorname{AppellF1} \left[ 2+m, \frac{3}{2}, -\frac{1}{2}, \right. \right. \right. \right. \\
 & \quad \left. \left. \left. 3+m, -i a x, i a x \right] + \operatorname{HypergeometricPFQ} \left[ \left\{ \frac{1}{2}, 1+\frac{m}{2} \right\}, \left\{ 2+\frac{m}{2} \right\}, -a^2 x^2 \right] \right) \right) \right)
 \end{aligned}$$

**Problem 143: Result unnecessarily involves higher level functions.**

$$\int e^{-3 i \operatorname{ArcTan}[a x]} x^m dx$$

Optimal (type 5, 159 leaves, 9 steps):

$$\begin{aligned}
 & - \frac{3 x^{1+m} \operatorname{Hypergeometric2F1} \left[ \frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, -a^2 x^2 \right]}{1+m} + \frac{i a x^{2+m} \operatorname{Hypergeometric2F1} \left[ \frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, -a^2 x^2 \right]}{2+m} + \\
 & \frac{4 x^{1+m} \operatorname{Hypergeometric2F1} \left[ \frac{3}{2}, \frac{1+m}{2}, \frac{3+m}{2}, -a^2 x^2 \right]}{1+m} - \frac{4 i a x^{2+m} \operatorname{Hypergeometric2F1} \left[ \frac{3}{2}, \frac{2+m}{2}, \frac{4+m}{2}, -a^2 x^2 \right]}{2+m}
 \end{aligned}$$

Result (type 6, 315 leaves):

$$\begin{aligned}
 & \left( 2 (2+m) x^{1+m} \sqrt{i+a x} \right. \\
 & \quad \left( - \left( \left( 2 \operatorname{AppellF1} \left[ 1+m, \frac{3}{2}, -\frac{1}{2}, 2+m, -i a x, i a x \right] \right) / \left( 2 (2+m) \operatorname{AppellF1} \left[ 1+m, \frac{3}{2}, -\frac{1}{2}, \right. \right. \right. \right. \right. \\
 & \quad \left. \left. \left. 2+m, -i a x, i a x \right] - i a x \left( \operatorname{AppellF1} \left[ 2+m, \frac{3}{2}, \frac{1}{2}, 3+m, -i a x, i a x \right] + \right. \right. \right. \\
 & \quad \left. \left. \left. 3 \operatorname{AppellF1} \left[ 2+m, \frac{5}{2}, -\frac{1}{2}, 3+m, -i a x, i a x \right] \right) \right) \right) + \\
 & \quad \left( i \sqrt{1+i a x} \sqrt{1+a^2 x^2} \operatorname{AppellF1} \left[ 1+m, \frac{1}{2}, -\frac{1}{2}, 2+m, -i a x, i a x \right] \right) / \\
 & \quad \left( \sqrt{1-i a x} \left( 2 i (2+m) \operatorname{AppellF1} \left[ 1+m, \frac{1}{2}, -\frac{1}{2}, 2+m, -i a x, i a x \right] + \right. \right. \\
 & \quad \left. \left. a x \left( \operatorname{AppellF1} \left[ 2+m, \frac{3}{2}, -\frac{1}{2}, 3+m, -i a x, i a x \right] + \right. \right. \right. \\
 & \quad \left. \left. \left. \operatorname{HypergeometricPFQ} \left[ \left\{ \frac{1}{2}, 1+\frac{m}{2} \right\}, \left\{ 2+\frac{m}{2} \right\}, -a^2 x^2 \right] \right) \right) \right) \right) / \left( (1+m) (-i+a x)^{3/2} \right)
 \end{aligned}$$

**Problem 144: Unable to integrate problem.**

$$\int e^{\frac{5}{2} i \operatorname{ArcTan}[a x]} x^m dx$$

Optimal (type 6, 36 leaves, 2 steps):



$$\frac{x^{1+m} \text{AppellF1}\left[1+m, \frac{5}{4}, -\frac{5}{4}, 2+m, i a x, -i a x\right]}{1+m}$$

Result (type 8, 18 leaves):

$$\int e^{\frac{5}{2} i \text{ArcTan}[a x]} x^m dx$$

**Problem 145: Unable to integrate problem.**

$$\int e^{\frac{3}{2} i \text{ArcTan}[a x]} x^m dx$$

Optimal (type 6, 36 leaves, 2 steps):

$$\frac{x^{1+m} \text{AppellF1}\left[1+m, \frac{3}{4}, -\frac{3}{4}, 2+m, i a x, -i a x\right]}{1+m}$$

Result (type 8, 18 leaves):

$$\int e^{\frac{3}{2} i \text{ArcTan}[a x]} x^m dx$$

**Problem 146: Unable to integrate problem.**

$$\int e^{\frac{1}{2} i \text{ArcTan}[a x]} x^m dx$$

Optimal (type 6, 36 leaves, 2 steps):

$$\frac{x^{1+m} \text{AppellF1}\left[1+m, \frac{1}{4}, -\frac{1}{4}, 2+m, i a x, -i a x\right]}{1+m}$$

Result (type 8, 18 leaves):

$$\int e^{\frac{1}{2} i \text{ArcTan}[a x]} x^m dx$$

**Problem 147: Unable to integrate problem.**

$$\int e^{-\frac{1}{2} i \text{ArcTan}[a x]} x^m dx$$

Optimal (type 6, 36 leaves, 2 steps):

$$\frac{x^{1+m} \text{AppellF1}\left[1+m, -\frac{1}{4}, \frac{1}{4}, 2+m, i a x, -i a x\right]}{1+m}$$

Result (type 8, 18 leaves):

$$\int e^{-\frac{1}{2} i \text{ArcTan}[a x]} x^m dx$$

### Problem 148: Unable to integrate problem.

$$\int e^{-\frac{3}{2} i \operatorname{ArcTan}[a x]} x^m dx$$

Optimal (type 6, 36 leaves, 2 steps):

$$\frac{x^{1+m} \operatorname{AppellF1}\left[1+m, -\frac{3}{4}, \frac{3}{4}, 2+m, i a x, -i a x\right]}{1+m}$$

Result (type 8, 18 leaves):

$$\int e^{-\frac{3}{2} i \operatorname{ArcTan}[a x]} x^m dx$$

### Problem 149: Unable to integrate problem.

$$\int e^{-\frac{5}{2} i \operatorname{ArcTan}[a x]} x^m dx$$

Optimal (type 6, 36 leaves, 2 steps):

$$\frac{x^{1+m} \operatorname{AppellF1}\left[1+m, -\frac{5}{4}, \frac{5}{4}, 2+m, i a x, -i a x\right]}{1+m}$$

Result (type 8, 18 leaves):

$$\int e^{-\frac{5}{2} i \operatorname{ArcTan}[a x]} x^m dx$$

### Problem 150: Unable to integrate problem.

$$\int e^{\frac{2 \operatorname{ArcTan}[x]}{3}} x^m dx$$

Optimal (type 6, 38 leaves, 2 steps):

$$\frac{x^{1+m} \operatorname{AppellF1}\left[1+m, -\frac{i}{3}, \frac{i}{3}, 2+m, i x, -i x\right]}{1+m}$$

Result (type 8, 14 leaves):

$$\int e^{\frac{2 \operatorname{ArcTan}[x]}{3}} x^m dx$$

### Problem 151: Unable to integrate problem.

$$\int e^{\frac{\operatorname{ArcTan}[x]}{3}} x^m dx$$

Optimal (type 6, 38 leaves, 2 steps):

$$\frac{x^{1+m} \operatorname{AppellF1}\left[1+m, -\frac{i}{6}, \frac{i}{6}, 2+m, i x, -i x\right]}{1+m}$$

Result (type 8, 14 leaves):

$$\int e^{\frac{\text{ArcTan}[x]}{3}} x^m dx$$

Problem 152: Unable to integrate problem.

$$\int e^{\frac{1}{4} i \text{ArcTan}[a x]} x^m dx$$

Optimal (type 6, 36 leaves, 2 steps):

$$\frac{x^{1+m} \text{AppellF1}\left[1+m, \frac{1}{8}, -\frac{1}{8}, 2+m, i a x, -i a x\right]}{1+m}$$

Result (type 8, 18 leaves):

$$\int e^{\frac{1}{4} i \text{ArcTan}[a x]} x^m dx$$

Problem 153: Unable to integrate problem.

$$\int e^{i n \text{ArcTan}[a x]} x^m dx$$

Optimal (type 6, 40 leaves, 2 steps):

$$\frac{x^{1+m} \text{AppellF1}\left[1+m, \frac{n}{2}, -\frac{n}{2}, 2+m, i a x, -i a x\right]}{1+m}$$

Result (type 8, 17 leaves):

$$\int e^{i n \text{ArcTan}[a x]} x^m dx$$

Problem 176: Result more than twice size of optimal antiderivative.

$$\int \frac{e^{2 i \text{ArcTan}[a+b x]}}{x} dx$$

Optimal (type 3, 38 leaves, 3 steps):

$$\frac{(i-a) \text{Log}[x]}{i+a} - \frac{2 \text{Log}[i+a+b x]}{1-i a}$$

Result (type 3, 125 leaves):

$$\frac{1}{2(i+a)} \left( (2+2i a) \text{ArcTan}\left[\frac{2a}{-1+e^{2i \text{ArcTan}[a+b x]} + a^2(1+e^{2i \text{ArcTan}[a+b x]}}}\right] + \right. \\ \left. 2(i+a) \text{Log}\left[1+e^{2i \text{ArcTan}[a+b x]}\right] - (-i+a) \text{Log}\left[(-1+e^{2i \text{ArcTan}[a+b x]})^2 + a^2(1+e^{2i \text{ArcTan}[a+b x]})^2\right] \right)$$

**Problem 203: Result more than twice size of optimal antiderivative.**

$$\int \frac{e^{-2i \operatorname{ArcTan}[a+bx]}}{x} dx$$

Optimal (type 3, 41 leaves, 3 steps):

$$\frac{(i+a) \operatorname{Log}[x]}{i-a} - \frac{2 \operatorname{Log}[i-a-bx]}{1+ia}$$

Result (type 3, 138 leaves):

$$\frac{1}{2(-i+a)} \left( (2-2ia) \operatorname{ArcTan}\left[\frac{2a}{-1+e^{-2i \operatorname{ArcTan}[a+bx]} + a^2(1+e^{-2i \operatorname{ArcTan}[a+bx]}}\right] + 2(-i+a) \operatorname{Log}[1+e^{-2i \operatorname{ArcTan}[a+bx]}] - (i+a) \operatorname{Log}\left[e^{-4i \operatorname{ArcTan}[a+bx]} \left( (-1+e^{2i \operatorname{ArcTan}[a+bx]})^2 + a^2(1+e^{2i \operatorname{ArcTan}[a+bx]})^2 \right) \right] \right)$$

**Problem 218: Result is not expressed in closed-form.**

$$\int e^{\frac{1}{2}i \operatorname{ArcTan}[a+bx]} dx$$

Optimal (type 3, 338 leaves, 13 steps):

$$\frac{i(1-ia-ibx)^{3/4}(1+ia+ibx)^{1/4}}{b} - \frac{i \operatorname{ArcTan}\left[1 - \frac{\sqrt{2}(1-ia-ibx)^{1/4}}{(1+ia+ibx)^{1/4}}\right]}{\sqrt{2}b} + \frac{i \operatorname{ArcTan}\left[1 + \frac{\sqrt{2}(1-ia-ibx)^{1/4}}{(1+ia+ibx)^{1/4}}\right]}{\sqrt{2}b} + \frac{i \operatorname{Log}\left[1 + \frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}} - \frac{\sqrt{2}(1-ia-ibx)^{1/4}}{(1+ia+ibx)^{1/4}}\right]}{2\sqrt{2}b} - \frac{i \operatorname{Log}\left[1 + \frac{\sqrt{1-ia-ibx}}{\sqrt{1+ia+ibx}} + \frac{\sqrt{2}(1-ia-ibx)^{1/4}}{(1+ia+ibx)^{1/4}}\right]}{2\sqrt{2}b}$$

Result (type 7, 87 leaves):

$$-\frac{1}{4b} \left( -\frac{8ie^{\frac{1}{2}i \operatorname{ArcTan}[a+bx]}}{1+e^{2i \operatorname{ArcTan}[a+bx]}} + \operatorname{RootSum}\left[1+\#1^4 \&, \frac{\operatorname{ArcTan}[a+bx] + 2i \operatorname{Log}\left[e^{\frac{1}{2}i \operatorname{ArcTan}[a+bx]} - \#1\right]}{\#1^3} \&\right] \right)$$

**Problem 219: Result is not expressed in closed-form.**

$$\int \frac{e^{\frac{1}{2}i \operatorname{ArcTan}[a+bx]}}{x} dx$$

Optimal (type 3, 395 leaves, 15 steps):

$$\begin{aligned}
 & - \frac{2 (\mathbf{i} - \mathbf{a})^{1/4} \operatorname{ArcTan} \left[ \frac{(\mathbf{i} + \mathbf{a})^{1/4} (1 + \mathbf{i} (\mathbf{a} + \mathbf{b} \mathbf{x}))^{1/4}}{(\mathbf{i} - \mathbf{a})^{1/4} (1 - \mathbf{i} (\mathbf{a} + \mathbf{b} \mathbf{x}))^{1/4}} \right]}{(\mathbf{i} + \mathbf{a})^{1/4}} - \sqrt{2} \operatorname{ArcTan} \left[ 1 - \frac{\sqrt{2} (1 + \mathbf{i} (\mathbf{a} + \mathbf{b} \mathbf{x}))^{1/4}}{(1 - \mathbf{i} (\mathbf{a} + \mathbf{b} \mathbf{x}))^{1/4}} \right] + \\
 & \sqrt{2} \operatorname{ArcTan} \left[ 1 + \frac{\sqrt{2} (1 + \mathbf{i} (\mathbf{a} + \mathbf{b} \mathbf{x}))^{1/4}}{(1 - \mathbf{i} (\mathbf{a} + \mathbf{b} \mathbf{x}))^{1/4}} \right] - \frac{2 (\mathbf{i} - \mathbf{a})^{1/4} \operatorname{ArcTanh} \left[ \frac{(\mathbf{i} + \mathbf{a})^{1/4} (1 + \mathbf{i} (\mathbf{a} + \mathbf{b} \mathbf{x}))^{1/4}}{(\mathbf{i} - \mathbf{a})^{1/4} (1 - \mathbf{i} (\mathbf{a} + \mathbf{b} \mathbf{x}))^{1/4}} \right]}{(\mathbf{i} + \mathbf{a})^{1/4}} - \\
 & \frac{\operatorname{Log} \left[ 1 - \frac{\sqrt{2} (1 + \mathbf{i} (\mathbf{a} + \mathbf{b} \mathbf{x}))^{1/4}}{(1 - \mathbf{i} (\mathbf{a} + \mathbf{b} \mathbf{x}))^{1/4}} + \frac{\sqrt{1 + \mathbf{i} (\mathbf{a} + \mathbf{b} \mathbf{x})}}{\sqrt{1 - \mathbf{i} (\mathbf{a} + \mathbf{b} \mathbf{x})}} \right]}{\sqrt{2}} + \frac{\operatorname{Log} \left[ 1 + \frac{\sqrt{2} (1 + \mathbf{i} (\mathbf{a} + \mathbf{b} \mathbf{x}))^{1/4}}{(1 - \mathbf{i} (\mathbf{a} + \mathbf{b} \mathbf{x}))^{1/4}} + \frac{\sqrt{1 + \mathbf{i} (\mathbf{a} + \mathbf{b} \mathbf{x})}}{\sqrt{1 - \mathbf{i} (\mathbf{a} + \mathbf{b} \mathbf{x})}} \right]}{\sqrt{2}}
 \end{aligned}$$

Result (type 7, 184 leaves):

$$\begin{aligned}
 & (-1)^{1/4} \left( -\operatorname{Log} \left[ (-1)^{1/4} - e^{\frac{1}{2} \mathbf{i} \operatorname{ArcTan}[\mathbf{a} + \mathbf{b} \mathbf{x}]} \right] - \mathbf{i} \operatorname{Log} \left[ (-1)^{3/4} - e^{\frac{1}{2} \mathbf{i} \operatorname{ArcTan}[\mathbf{a} + \mathbf{b} \mathbf{x}]} \right] \right) + \\
 & \operatorname{Log} \left[ (-1)^{1/4} + e^{\frac{1}{2} \mathbf{i} \operatorname{ArcTan}[\mathbf{a} + \mathbf{b} \mathbf{x}]} \right] + \mathbf{i} \operatorname{Log} \left[ (-1)^{3/4} + e^{\frac{1}{2} \mathbf{i} \operatorname{ArcTan}[\mathbf{a} + \mathbf{b} \mathbf{x}]} \right] \right) + \frac{1}{2 (\mathbf{i} + \mathbf{a})} \\
 & (\mathbf{i} + \mathbf{a}) \operatorname{RootSum} \left[ -\mathbf{i} + \mathbf{a} + \mathbf{i} \#1^4 + \mathbf{a} \#1^4 \&, \frac{\operatorname{ArcTan}[\mathbf{a} + \mathbf{b} \mathbf{x}] + \mathbf{i} \operatorname{Log} \left[ \left( e^{\frac{1}{2} \mathbf{i} \operatorname{ArcTan}[\mathbf{a} + \mathbf{b} \mathbf{x}]} - \#1 \right)^2 \right]}{\#1^3} \& \right]
 \end{aligned}$$

Problem 220: Result is not expressed in closed-form.

$$\int \frac{e^{\frac{1}{2} \mathbf{i} \operatorname{ArcTan}[\mathbf{a} + \mathbf{b} \mathbf{x}]}}{x^2} dx$$

Optimal (type 3, 205 leaves, 6 steps):

$$\begin{aligned}
 & - \frac{(\mathbf{i} + \mathbf{a} + \mathbf{b} \mathbf{x}) (1 + \mathbf{i} (\mathbf{a} + \mathbf{b} \mathbf{x}))^{1/4}}{(\mathbf{i} + \mathbf{a}) \mathbf{x} (1 - \mathbf{i} (\mathbf{a} + \mathbf{b} \mathbf{x}))^{1/4}} + \\
 & \frac{\mathbf{i} \mathbf{b} \operatorname{ArcTan} \left[ \frac{(\mathbf{i} + \mathbf{a})^{1/4} (1 + \mathbf{i} (\mathbf{a} + \mathbf{b} \mathbf{x}))^{1/4}}{(\mathbf{i} - \mathbf{a})^{1/4} (1 - \mathbf{i} (\mathbf{a} + \mathbf{b} \mathbf{x}))^{1/4}} \right]}{(\mathbf{i} - \mathbf{a})^{3/4} (\mathbf{i} + \mathbf{a})^{5/4}} + \frac{\mathbf{i} \mathbf{b} \operatorname{ArcTanh} \left[ \frac{(\mathbf{i} + \mathbf{a})^{1/4} (1 + \mathbf{i} (\mathbf{a} + \mathbf{b} \mathbf{x}))^{1/4}}{(\mathbf{i} - \mathbf{a})^{1/4} (1 - \mathbf{i} (\mathbf{a} + \mathbf{b} \mathbf{x}))^{1/4}} \right]}{(\mathbf{i} - \mathbf{a})^{3/4} (\mathbf{i} + \mathbf{a})^{5/4}}
 \end{aligned}$$

Result (type 7, 131 leaves):

$$\begin{aligned}
 & - \frac{1}{4 (\mathbf{i} + \mathbf{a})^2} \mathbf{b} \left( \frac{8 (\mathbf{i} + \mathbf{a}) e^{\frac{1}{2} \mathbf{i} \operatorname{ArcTan}[\mathbf{a} + \mathbf{b} \mathbf{x}]}}{1 - e^{2 \mathbf{i} \operatorname{ArcTan}[\mathbf{a} + \mathbf{b} \mathbf{x}]} + \mathbf{i} \mathbf{a} (1 + e^{2 \mathbf{i} \operatorname{ArcTan}[\mathbf{a} + \mathbf{b} \mathbf{x}])} + \right. \\
 & \left. \operatorname{RootSum} \left[ -\mathbf{i} + \mathbf{a} + \mathbf{i} \#1^4 + \mathbf{a} \#1^4 \&, \frac{\operatorname{ArcTan}[\mathbf{a} + \mathbf{b} \mathbf{x}] + \mathbf{i} \operatorname{Log} \left[ \left( e^{\frac{1}{2} \mathbf{i} \operatorname{ArcTan}[\mathbf{a} + \mathbf{b} \mathbf{x}]} - \#1 \right)^2 \right]}{\#1^3} \& \right] \right)
 \end{aligned}$$

Problem 223: Result is not expressed in closed-form.

$$\int e^{\frac{3}{2} \mathbf{i} \operatorname{ArcTan}[\mathbf{a} + \mathbf{b} \mathbf{x}]} dx$$

Optimal (type 3, 338 leaves, 13 steps):

$$\frac{i (1 - i a - i b x)^{1/4} (1 + i a + i b x)^{3/4}}{b} - \frac{3 i \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} (1 - i a - i b x)^{1/4}}{(1 + i a + i b x)^{1/4}}\right]}{\sqrt{2} b} + \frac{3 i \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} (1 - i a - i b x)^{1/4}}{(1 + i a + i b x)^{1/4}}\right]}{\sqrt{2} b} - \frac{3 i \operatorname{Log}\left[1 + \frac{\sqrt{1 - i a - i b x}}{\sqrt{1 + i a + i b x}} - \frac{\sqrt{2} (1 - i a - i b x)^{1/4}}{(1 + i a + i b x)^{1/4}}\right]}{2 \sqrt{2} b} + \frac{3 i \operatorname{Log}\left[1 + \frac{\sqrt{1 - i a - i b x}}{\sqrt{1 + i a + i b x}} + \frac{\sqrt{2} (1 - i a - i b x)^{1/4}}{(1 + i a + i b x)^{1/4}}\right]}{2 \sqrt{2} b}$$

Result (type 7, 90 leaves):

$$\frac{2 i e^{\frac{3}{2} i \operatorname{ArcTan}[a + b x]}}{b (1 + e^{2 i \operatorname{ArcTan}[a + b x]})} - \frac{3 \operatorname{RootSum}\left[1 + \#1^4 \&, \frac{\operatorname{ArcTan}[a + b x] + 2 i \operatorname{Log}\left[e^{\frac{1}{2} i \operatorname{ArcTan}[a + b x]} - \#1\right]}{\#1}\right] \&}{4 b}$$

Problem 224: Result is not expressed in closed-form.

$$\int \frac{e^{\frac{3}{2} i \operatorname{ArcTan}[a + b x]}}{x} dx$$

Optimal (type 3, 427 leaves, 18 steps):

$$\frac{2 (i - a)^{3/4} \operatorname{ArcTan}\left[\frac{(i + a)^{1/4} (1 + i a + i b x)^{1/4}}{(i - a)^{1/4} (1 - i a - i b x)^{1/4}}\right]}{(i + a)^{3/4}} + \sqrt{2} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} (1 - i a - i b x)^{1/4}}{(1 + i a + i b x)^{1/4}}\right] - \sqrt{2} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} (1 - i a - i b x)^{1/4}}{(1 + i a + i b x)^{1/4}}\right] - \frac{2 (i - a)^{3/4} \operatorname{ArcTanh}\left[\frac{(i + a)^{1/4} (1 + i a + i b x)^{1/4}}{(i - a)^{1/4} (1 - i a - i b x)^{1/4}}\right]}{(i + a)^{3/4}} + \frac{\operatorname{Log}\left[1 + \frac{\sqrt{1 - i a - i b x}}{\sqrt{1 + i a + i b x}} - \frac{\sqrt{2} (1 - i a - i b x)^{1/4}}{(1 + i a + i b x)^{1/4}}\right]}{\sqrt{2}} - \frac{\operatorname{Log}\left[1 + \frac{\sqrt{1 - i a - i b x}}{\sqrt{1 + i a + i b x}} + \frac{\sqrt{2} (1 - i a - i b x)^{1/4}}{(1 + i a + i b x)^{1/4}}\right]}{\sqrt{2}}$$

Result (type 7, 184 leaves):

$$(-1)^{1/4} \left( -i \operatorname{Log}\left[(-1)^{1/4} - e^{\frac{1}{2} i \operatorname{ArcTan}[a + b x]}\right] - \operatorname{Log}\left[(-1)^{3/4} - e^{\frac{1}{2} i \operatorname{ArcTan}[a + b x]}\right] + i \operatorname{Log}\left[(-1)^{1/4} + e^{\frac{1}{2} i \operatorname{ArcTan}[a + b x]}\right] + \operatorname{Log}\left[(-1)^{3/4} + e^{\frac{1}{2} i \operatorname{ArcTan}[a + b x]}\right] \right) + \frac{1}{2 (i + a)} (1 + i a) \operatorname{RootSum}\left[-i + a + i \#1^4 + a \#1^4 \&, \frac{\operatorname{ArcTan}[a + b x] + i \operatorname{Log}\left[\left(e^{\frac{1}{2} i \operatorname{ArcTan}[a + b x]} - \#1\right)^2\right]}{\#1}\right] \&$$

Problem 225: Result is not expressed in closed-form.

$$\int \frac{e^{\frac{3}{2} i \operatorname{ArcTan}[a + b x]}}{x^2} dx$$

Optimal (type 3, 211 leaves, 6 steps):

$$\begin{aligned}
 & - \frac{(1 - i a - i b x)^{1/4} (1 + i a + i b x)^{3/4}}{(1 - i a) x} - \\
 & \frac{3 i b \operatorname{ArcTan}\left[\frac{(i+a)^{1/4} (1+i a+i b x)^{1/4}}{(i-a)^{1/4} (1-i a-i b x)^{1/4}}\right]}{(i-a)^{1/4} (i+a)^{7/4}} + \frac{3 i b \operatorname{ArcTanh}\left[\frac{(i+a)^{1/4} (1+i a+i b x)^{1/4}}{(i-a)^{1/4} (1-i a-i b x)^{1/4}}\right]}{(i-a)^{1/4} (i+a)^{7/4}}
 \end{aligned}$$

Result (type 7, 131 leaves):

$$\begin{aligned}
 & \frac{1}{4 (i+a)^2} b \left( \frac{8 (i+a) e^{\frac{3}{2} i \operatorname{ArcTan}[a+b x]}}{-1 + e^{2 i \operatorname{ArcTan}[a+b x]} - i a (1 + e^{2 i \operatorname{ArcTan}[a+b x]})} - \right. \\
 & \left. 3 \operatorname{RootSum}\left[-i + a + i \#1^4 + a \#1^4 \&, \frac{\operatorname{ArcTan}[a+b x] + i \operatorname{Log}\left[\left(e^{\frac{1}{2} i \operatorname{ArcTan}[a+b x]} - \#1\right)^2\right]}{\#1}\right] \& \right)
 \end{aligned}$$

**Problem 228: Result is not expressed in closed-form.**

$$\int e^{-\frac{1}{2} i \operatorname{ArcTan}[a+b x]} dx$$

Optimal (type 3, 338 leaves, 13 steps):

$$\begin{aligned}
 & - \frac{i (1 - i a - i b x)^{1/4} (1 + i a + i b x)^{3/4}}{b} - \\
 & \frac{i \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} (1-i a-i b x)^{1/4}}{(1+i a+i b x)^{1/4}}\right]}{\sqrt{2} b} + \frac{i \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} (1-i a-i b x)^{1/4}}{(1+i a+i b x)^{1/4}}\right]}{\sqrt{2} b} - \\
 & \frac{i \operatorname{Log}\left[1 + \frac{\sqrt{1-i a-i b x}}{\sqrt{1+i a+i b x}} - \frac{\sqrt{2} (1-i a-i b x)^{1/4}}{(1+i a+i b x)^{1/4}}\right]}{2 \sqrt{2} b} + \frac{i \operatorname{Log}\left[1 + \frac{\sqrt{1-i a-i b x}}{\sqrt{1+i a+i b x}} + \frac{\sqrt{2} (1-i a-i b x)^{1/4}}{(1+i a+i b x)^{1/4}}\right]}{2 \sqrt{2} b}
 \end{aligned}$$

Result (type 7, 89 leaves):

$$\frac{1}{4 b} \left( - \frac{8 i e^{\frac{3}{2} i \operatorname{ArcTan}[a+b x]}}{1 + e^{2 i \operatorname{ArcTan}[a+b x]}} + \operatorname{RootSum}\left[1 + \#1^4 \&, \frac{-\operatorname{ArcTan}[a+b x] + 2 i \operatorname{Log}\left[e^{-\frac{1}{2} i \operatorname{ArcTan}[a+b x]} - \#1\right]}{\#1^3}\right] \& \right)$$

**Problem 229: Result is not expressed in closed-form.**

$$\int \frac{e^{-\frac{1}{2} i \operatorname{ArcTan}[a+b x]}}{x} dx$$

Optimal (type 3, 395 leaves, 14 steps):

$$\begin{aligned}
 & - \frac{2 (i + a)^{1/4} \operatorname{ArcTan}\left[\frac{(i-a)^{1/4} (1-i(a+bx))^{1/4}}{(i+a)^{1/4} (1+i(a+bx))^{1/4}}\right]}{(i-a)^{1/4}} - \sqrt{2} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} (1-i(a+bx))^{1/4}}{(1+i(a+bx))^{1/4}}\right] + \\
 & \sqrt{2} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} (1-i(a+bx))^{1/4}}{(1+i(a+bx))^{1/4}}\right] - \frac{2 (i + a)^{1/4} \operatorname{ArcTanh}\left[\frac{(i-a)^{1/4} (1-i(a+bx))^{1/4}}{(i+a)^{1/4} (1+i(a+bx))^{1/4}}\right]}{(i-a)^{1/4}} - \\
 & \frac{\operatorname{Log}\left[1 + \frac{\sqrt{1-i(a+bx)}}{\sqrt{1+i(a+bx)}} - \frac{\sqrt{2} (1-i(a+bx))^{1/4}}{(1+i(a+bx))^{1/4}}\right]}{\sqrt{2}} + \frac{\operatorname{Log}\left[1 + \frac{\sqrt{1-i(a+bx)}}{\sqrt{1+i(a+bx)}} + \frac{\sqrt{2} (1-i(a+bx))^{1/4}}{(1+i(a+bx))^{1/4}}\right]}{\sqrt{2}}
 \end{aligned}$$

Result (type 7, 236 leaves):

$$\begin{aligned}
 & (-1)^{1/4} \left( i \operatorname{Log}\left[ e^{-2i \operatorname{ArcTan}[a+bx]} \left( (-1)^{1/4} - e^{\frac{1}{2}i \operatorname{ArcTan}[a+bx]} \right) \right] + \right. \\
 & \operatorname{Log}\left[ e^{-2i \operatorname{ArcTan}[a+bx]} \left( (-1)^{3/4} - e^{\frac{1}{2}i \operatorname{ArcTan}[a+bx]} \right) \right] - \\
 & i \operatorname{Log}\left[ e^{-2i \operatorname{ArcTan}[a+bx]} \left( (-1)^{1/4} + e^{\frac{1}{2}i \operatorname{ArcTan}[a+bx]} \right) \right] - \\
 & \left. \operatorname{Log}\left[ e^{-2i \operatorname{ArcTan}[a+bx]} \left( (-1)^{3/4} + e^{\frac{1}{2}i \operatorname{ArcTan}[a+bx]} \right) \right] \right) + \frac{1}{2(-i+a)} \\
 & (1-i a) \operatorname{RootSum}\left[ i + a - i \#1^4 + a \#1^4 \&, \frac{\operatorname{ArcTan}[a+bx] - i \operatorname{Log}\left[ \left( e^{-\frac{1}{2}i \operatorname{ArcTan}[a+bx]} - \#1 \right)^2 \right]}{\#1^3} \& \right]
 \end{aligned}$$

Problem 230: Result is not expressed in closed-form.

$$\int \frac{e^{-\frac{1}{2}i \operatorname{ArcTan}[a+bx]}}{x^2} dx$$

Optimal (type 3, 210 leaves, 5 steps):

$$\begin{aligned}
 & - \frac{(i-a-bx)(1-i(a+bx))^{1/4}}{(i-a)x(1+i(a+bx))^{1/4}} - \\
 & \frac{i b \operatorname{ArcTan}\left[\frac{(i-a)^{1/4} (1-i(a+bx))^{1/4}}{(i+a)^{1/4} (1+i(a+bx))^{1/4}}\right]}{(i-a)^{5/4} (i+a)^{3/4}} - \frac{i b \operatorname{ArcTanh}\left[\frac{(i-a)^{1/4} (1-i(a+bx))^{1/4}}{(i+a)^{1/4} (1+i(a+bx))^{1/4}}\right]}{(i-a)^{5/4} (i+a)^{3/4}}
 \end{aligned}$$

Result (type 7, 133 leaves):

$$\begin{aligned}
 & \frac{1}{4(-i+a)^2} b \left( \frac{8(-i+a) e^{\frac{3}{2}i \operatorname{ArcTan}[a+bx]}}{1 - e^{2i \operatorname{ArcTan}[a+bx]} + i a (1 + e^{2i \operatorname{ArcTan}[a+bx]})} + \right. \\
 & \left. \operatorname{RootSum}\left[ i + a - i \#1^4 + a \#1^4 \&, \frac{-\operatorname{ArcTan}[a+bx] + i \operatorname{Log}\left[ \left( e^{-\frac{1}{2}i \operatorname{ArcTan}[a+bx]} - \#1 \right)^2 \right]}{\#1^3} \& \right] \right)
 \end{aligned}$$



### Problem 233: Result is not expressed in closed-form.

$$\int e^{-\frac{3}{2}i \operatorname{ArcTan}[a+bx]} dx$$

Optimal (type 3, 338 leaves, 13 steps):

$$\begin{aligned} & -\frac{i(1-i a-i b x)^{3/4}(1+i a+i b x)^{1/4}}{b} \\ & -\frac{3i \operatorname{ArcTan}\left[1-\frac{\sqrt{2}(1-i a-i b x)^{1/4}}{(1+i a+i b x)^{1/4}}\right]}{\sqrt{2} b} + \frac{3i \operatorname{ArcTan}\left[1+\frac{\sqrt{2}(1-i a-i b x)^{1/4}}{(1+i a+i b x)^{1/4}}\right]}{\sqrt{2} b} + \\ & -\frac{3i \operatorname{Log}\left[1+\frac{\sqrt{1-i a-i b x}}{\sqrt{1+i a+i b x}}-\frac{\sqrt{2}(1-i a-i b x)^{1/4}}{(1+i a+i b x)^{1/4}}\right]}{2\sqrt{2} b} - \frac{3i \operatorname{Log}\left[1+\frac{\sqrt{1-i a-i b x}}{\sqrt{1+i a+i b x}}+\frac{\sqrt{2}(1-i a-i b x)^{1/4}}{(1+i a+i b x)^{1/4}}\right]}{2\sqrt{2} b} \end{aligned}$$

Result (type 7, 90 leaves):

$$-\frac{2i e^{-\frac{3}{2}i \operatorname{ArcTan}[a+bx]}}{b(1+e^{-2i \operatorname{ArcTan}[a+bx]})} - \frac{3 \operatorname{RootSum}\left[1+\#1^4 \&, \frac{\operatorname{ArcTan}[a+bx]-2i \operatorname{Log}\left[e^{-\frac{1}{2}i \operatorname{ArcTan}[a+bx]}-\#1\right]}{\#1}\right] \&}{4b}$$

### Problem 234: Result is not expressed in closed-form.

$$\int \frac{e^{-\frac{3}{2}i \operatorname{ArcTan}[a+bx]}}{x} dx$$

Optimal (type 3, 427 leaves, 18 steps):

$$\begin{aligned} & -\frac{2(i+a)^{3/4} \operatorname{ArcTan}\left[\frac{(i+a)^{1/4}(1+i a+i b x)^{1/4}}{(i-a)^{1/4}(1-i a-i b x)^{1/4}}\right]}{(i-a)^{3/4}} - \sqrt{2} \operatorname{ArcTan}\left[1-\frac{\sqrt{2}(1-i a-i b x)^{1/4}}{(1+i a+i b x)^{1/4}}\right] + \\ & \sqrt{2} \operatorname{ArcTan}\left[1+\frac{\sqrt{2}(1-i a-i b x)^{1/4}}{(1+i a+i b x)^{1/4}}\right] - \frac{2(i+a)^{3/4} \operatorname{ArcTanh}\left[\frac{(i+a)^{1/4}(1+i a+i b x)^{1/4}}{(i-a)^{1/4}(1-i a-i b x)^{1/4}}\right]}{(i-a)^{3/4}} + \\ & \frac{\operatorname{Log}\left[1+\frac{\sqrt{1-i a-i b x}}{\sqrt{1+i a+i b x}}-\frac{\sqrt{2}(1-i a-i b x)^{1/4}}{(1+i a+i b x)^{1/4}}\right]}{\sqrt{2}} - \frac{\operatorname{Log}\left[1+\frac{\sqrt{1-i a-i b x}}{\sqrt{1+i a+i b x}}+\frac{\sqrt{2}(1-i a-i b x)^{1/4}}{(1+i a+i b x)^{1/4}}\right]}{\sqrt{2}} \end{aligned}$$

Result (type 7, 237 leaves):

$$\begin{aligned}
 & (-1)^{1/4} \left( \text{Log} \left[ e^{-2 i \text{ArcTan}[a+b x]} \left( (-1)^{1/4} - e^{\frac{1}{2} i \text{ArcTan}[a+b x]} \right) \right] \right) + \\
 & i \left( \text{Log} \left[ e^{-2 i \text{ArcTan}[a+b x]} \left( (-1)^{3/4} - e^{\frac{1}{2} i \text{ArcTan}[a+b x]} \right) \right] \right) + \\
 & i \text{Log} \left[ e^{-2 i \text{ArcTan}[a+b x]} \left( (-1)^{1/4} + e^{\frac{1}{2} i \text{ArcTan}[a+b x]} \right) \right] - \\
 & \text{Log} \left[ e^{-2 i \text{ArcTan}[a+b x]} \left( (-1)^{3/4} + e^{\frac{1}{2} i \text{ArcTan}[a+b x]} \right) \right] \right) + \frac{1}{2(-i+a)} \\
 & (1-i a) \text{RootSum} \left[ i+a-i \#1^4+a \#1^4 \&, \frac{\text{ArcTan}[a+b x]-i \text{Log} \left[ \left( e^{-\frac{1}{2} i \text{ArcTan}[a+b x]} - \#1 \right)^2 \right]}{\#1} \& \right]
 \end{aligned}$$

Problem 235: Result is not expressed in closed-form.

$$\int \frac{e^{-\frac{3}{2} i \text{ArcTan}[a+b x]}}{x^2} dx$$

Optimal (type 3, 211 leaves, 6 steps):

$$\begin{aligned}
 & \frac{(1-i a-i b x)^{3/4} (1+i a+i b x)^{1/4}}{(1+i a) x} - \\
 & \frac{3 i b \text{ArcTan} \left[ \frac{(i+a)^{1/4} (1+i a+i b x)^{1/4}}{(i-a)^{1/4} (1-i a-i b x)^{1/4}} \right]}{(i-a)^{7/4} (i+a)^{1/4}} - \frac{3 i b \text{ArcTanh} \left[ \frac{(i+a)^{1/4} (1+i a+i b x)^{1/4}}{(i-a)^{1/4} (1-i a-i b x)^{1/4}} \right]}{(i-a)^{7/4} (i+a)^{1/4}}
 \end{aligned}$$

Result (type 7, 133 leaves):

$$\begin{aligned}
 & \frac{1}{4(-i+a)^2} b \left( \frac{8(-i+a) e^{\frac{1}{2} i \text{ArcTan}[a+b x]}}{1-e^{2 i \text{ArcTan}[a+b x]}+i a(1+e^{2 i \text{ArcTan}[a+b x]})} - \right. \\
 & \left. 3 \text{RootSum} \left[ i+a-i \#1^4+a \#1^4 \&, \frac{\text{ArcTan}[a+b x]-i \text{Log} \left[ \left( e^{-\frac{1}{2} i \text{ArcTan}[a+b x]} - \#1 \right)^2 \right]}{\#1} \& \right] \right)
 \end{aligned}$$

Problem 236: Unable to integrate problem.

$$\int e^{n \text{ArcTan}[a+b x]} x^m dx$$

Optimal (type 6, 140 leaves, 4 steps):

$$\begin{aligned}
 & \frac{1}{1+m} x^{1+m} (1-i a-i b x)^{\frac{i n}{2}} (1+i a+i b x)^{-\frac{i n}{2}} \left( 1 - \frac{b x}{i-a} \right)^{\frac{i n}{2}} \\
 & \left( 1 + \frac{b x}{i+a} \right)^{-\frac{i n}{2}} \text{AppellF1} \left[ 1+m, -\frac{i n}{2}, \frac{i n}{2}, 2+m, -\frac{b x}{i+a}, \frac{b x}{i-a} \right]
 \end{aligned}$$

Result (type 8, 16 leaves):

$$\int e^{n \operatorname{ArcTan}[a+bx]} x^m dx$$

**Problem 241: Unable to integrate problem.**

$$\int \frac{e^{n \operatorname{ArcTan}[a+bx]}}{x} dx$$

Optimal (type 5, 191 leaves, 5 steps):

$$\frac{1}{n} 2^{\frac{i n}{2}} (1 - i a - i b x)^{\frac{i n}{2}} (1 + i a + i b x)^{-\frac{i n}{2}}$$

$$\operatorname{Hypergeometric2F1}\left[1, \frac{i n}{2}, 1 + \frac{i n}{2}, \frac{(i - a)(1 - i a - i b x)}{(i + a)(1 + i a + i b x)}\right] - \frac{1}{n}$$

$$i 2^{1 - \frac{i n}{2}} (1 - i a - i b x)^{\frac{i n}{2}} \operatorname{Hypergeometric2F1}\left[\frac{i n}{2}, \frac{i n}{2}, 1 + \frac{i n}{2}, \frac{1}{2} (1 - i a - i b x)\right]$$

Result (type 8, 16 leaves):

$$\int \frac{e^{n \operatorname{ArcTan}[a+bx]}}{x} dx$$

**Problem 242: Unable to integrate problem.**

$$\int \frac{e^{n \operatorname{ArcTan}[a+bx]}}{x^2} dx$$

Optimal (type 5, 128 leaves, 2 steps):

$$- \left( \left( 4 b (1 - i a - i b x)^{1 + \frac{i n}{2}} (1 + i a + i b x)^{-1 - \frac{i n}{2}} \right. \right.$$

$$\left. \left. \operatorname{Hypergeometric2F1}\left[2, 1 + \frac{i n}{2}, 2 + \frac{i n}{2}, \frac{(i - a)(1 - i a - i b x)}{(i + a)(1 + i a + i b x)}\right] \right) / \left( (i + a)^2 (2 i - n) \right) \right)$$

Result (type 8, 16 leaves):

$$\int \frac{e^{n \operatorname{ArcTan}[a+bx]}}{x^2} dx$$

**Problem 243: Unable to integrate problem.**

$$\int \frac{e^{n \operatorname{ArcTan}[a+bx]}}{x^3} dx$$

Optimal (type 5, 207 leaves, 3 steps):

$$- \frac{(1 - i a - i b x)^{1 + \frac{i n}{2}} (1 + i a + i b x)^{1 - \frac{i n}{2}}}{2 (1 + a^2) x^2} - \left( 2 b^2 (2 a - n) (1 - i a - i b x)^{1 + \frac{i n}{2}} (1 + i a + i b x)^{-1 - \frac{i n}{2}} \right.$$

$$\left. \operatorname{Hypergeometric2F1}\left[2, 1 + \frac{i n}{2}, 2 + \frac{i n}{2}, \frac{(i - a)(1 - i a - i b x)}{(i + a)(1 + i a + i b x)}\right] \right) / \left( (i - a) (i + a)^3 (2 i - n) \right)$$

Result (type 8, 16 leaves):

$$\int \frac{e^{n \operatorname{ArcTan}[a+bx]}}{x^3} dx$$

**Problem 244: Unable to integrate problem.**

$$\int e^{\operatorname{ArcTan}[ax]} (c + a^2 c x^2)^p dx$$

Optimal (type 5, 102 leaves, 3 steps):

$$\frac{1}{a \left( (2+i) + 2p \right)} {}_2F_1 \left[ \frac{i}{2} - p, \left( 1 + \frac{i}{2} \right) + p, \left( 2 + \frac{i}{2} \right) + p, \frac{1}{2} (1 - i a x) \right]$$

Result (type 8, 21 leaves):

$$\int e^{\operatorname{ArcTan}[ax]} (c + a^2 c x^2)^p dx$$

**Problem 259: Unable to integrate problem.**

$$\int e^{2 \operatorname{ArcTan}[ax]} (c + a^2 c x^2)^p dx$$

Optimal (type 5, 90 leaves, 3 steps):

$$\frac{1}{a \left( (1+i) + p \right)} {}_2F_1 \left[ i - p, (1+i) + p, (2+i) + p, \frac{1}{2} (1 - i a x) \right]$$

Result (type 8, 23 leaves):

$$\int e^{2 \operatorname{ArcTan}[ax]} (c + a^2 c x^2)^p dx$$

**Problem 260: Result more than twice size of optimal antiderivative.**

$$\int e^{2 \operatorname{ArcTan}[ax]} (c + a^2 c x^2)^2 dx$$

Optimal (type 5, 53 leaves, 2 steps):

$$\frac{1}{a} \left( \frac{1}{5} + \frac{3i}{5} \right) 2^{1-i} c^2 (1 - i a x)^{3+i} {}_2F_1 \left[ -2+i, 3+i, 4+i, \frac{1}{2} (1 - i a x) \right]$$

Result (type 5, 114 leaves):

$$\frac{1}{30 a} c^2 e^{2 \operatorname{ArcTan}[a x]} \left( -13 + 56 a x - 16 a^2 x^2 + 22 a^3 x^3 - 3 a^4 x^4 + 6 a^5 x^5 - 40 i \operatorname{Hypergeometric2F1}\left[-i, 1, 1-i, -e^{2 i \operatorname{ArcTan}[a x]}\right] + (20 + 20 i) e^{2 i \operatorname{ArcTan}[a x]} \operatorname{Hypergeometric2F1}\left[1, 1-i, 2-i, -e^{2 i \operatorname{ArcTan}[a x]}\right] \right)$$

**Problem 273: Unable to integrate problem.**

$$\int e^{-\operatorname{ArcTan}[a x]} (c + a^2 c x^2)^p dx$$

Optimal (type 5, 101 leaves, 3 steps):

$$\left( 2^{\left(1+\frac{i}{2}\right)+p} (1-i a x)^{\left(1-\frac{i}{2}\right)+p} (1+a^2 x^2)^{-p} (c+a^2 c x^2)^p \operatorname{Hypergeometric2F1}\left[-\frac{i}{2}-p, \left(1-\frac{i}{2}\right)+p, \left(2-\frac{i}{2}\right)+p, \frac{1}{2}(1-i a x)\right] \right) / (a((-1-2i)-2ip))$$

Result (type 8, 23 leaves):

$$\int e^{-\operatorname{ArcTan}[a x]} (c + a^2 c x^2)^p dx$$

**Problem 283: Unable to integrate problem.**

$$\int \frac{e^{-\operatorname{ArcTan}[a x]}}{\sqrt{c + a^2 c x^2}} dx$$

Optimal (type 5, 93 leaves, 3 steps):

$$-\frac{1}{a \sqrt{c + a^2 c x^2}} (1-i) 2^{-\frac{1}{2}+\frac{i}{2}} (1-i a x)^{\frac{1}{2}-\frac{i}{2}} \sqrt{1+a^2 x^2} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}-\frac{i}{2}, \frac{1}{2}-\frac{i}{2}, \frac{3}{2}-\frac{i}{2}, \frac{1}{2}(1-i a x)\right]$$

Result (type 8, 25 leaves):

$$\int \frac{e^{-\operatorname{ArcTan}[a x]}}{\sqrt{c + a^2 c x^2}} dx$$

**Problem 284: Unable to integrate problem.**

$$\int \frac{e^{-\operatorname{ArcTan}[a x]}}{(c + a^2 c x^2)^{3/2}} dx$$

Optimal (type 3, 38 leaves, 1 step):

$$-\frac{e^{-\operatorname{ArcTan}[a x]} (1-a x)}{2 a c \sqrt{c + a^2 c x^2}}$$

Result (type 8, 25 leaves):

$$\int \frac{e^{-\text{ArcTan}[a x]}}{(c + a^2 c x^2)^{3/2}} dx$$

Problem 285: Unable to integrate problem.

$$\int \frac{e^{-\text{ArcTan}[a x]}}{(c + a^2 c x^2)^{5/2}} dx$$

Optimal (type 3, 77 leaves, 2 steps):

$$-\frac{e^{-\text{ArcTan}[a x]} (1 - 3 a x)}{10 a c (c + a^2 c x^2)^{3/2}} - \frac{3 e^{-\text{ArcTan}[a x]} (1 - a x)}{10 a c^2 \sqrt{c + a^2 c x^2}}$$

Result (type 8, 25 leaves):

$$\int \frac{e^{-\text{ArcTan}[a x]}}{(c + a^2 c x^2)^{5/2}} dx$$

Problem 286: Unable to integrate problem.

$$\int \frac{e^{-\text{ArcTan}[a x]}}{(c + a^2 c x^2)^{7/2}} dx$$

Optimal (type 3, 115 leaves, 3 steps):

$$-\frac{e^{-\text{ArcTan}[a x]} (1 - 5 a x)}{26 a c (c + a^2 c x^2)^{5/2}} - \frac{e^{-\text{ArcTan}[a x]} (1 - 3 a x)}{13 a c^2 (c + a^2 c x^2)^{3/2}} - \frac{3 e^{-\text{ArcTan}[a x]} (1 - a x)}{13 a c^3 \sqrt{c + a^2 c x^2}}$$

Result (type 8, 25 leaves):

$$\int \frac{e^{-\text{ArcTan}[a x]}}{(c + a^2 c x^2)^{7/2}} dx$$

Problem 287: Unable to integrate problem.

$$\int e^{-2 \text{ArcTan}[a x]} (c + a^2 c x^2)^p dx$$

Optimal (type 5, 90 leaves, 3 steps):

$$\frac{1}{a ((1 - i) + p)} i 2^{i+p} (1 - i a x)^{(1-i)+p} (1 + a^2 x^2)^{-p} (c + a^2 c x^2)^p$$

$$\text{Hypergeometric2F1}[-i - p, (1 - i) + p, (2 - i) + p, \frac{1}{2} (1 - i a x)]$$

Result (type 8, 23 leaves):

$$\int e^{-2 \text{ArcTan}[a x]} (c + a^2 c x^2)^p dx$$

### Problem 288: Result more than twice size of optimal antiderivative.

$$\int e^{-2 \operatorname{ArcTan}[a x]} (c + a^2 c x^2)^2 dx$$

Optimal (type 5, 53 leaves, 2 steps):

$$-\frac{1}{a} \left( \frac{1}{5} - \frac{3i}{5} \right) 2^{1+i} c^2 (1 - ia x)^{3-i} \operatorname{Hypergeometric2F1} \left[ -2 - i, 3 - i, 4 - i, \frac{1}{2} (1 - ia x) \right]$$

Result (type 5, 114 leaves):

$$\frac{1}{30a} c^2 e^{-2 \operatorname{ArcTan}[a x]} (13 + 56 a x + 16 a^2 x^2 + 22 a^3 x^3 + 3 a^4 x^4 + 6 a^5 x^5 - 40 i \operatorname{Hypergeometric2F1} [i, 1, 1 + i, -e^{2i \operatorname{ArcTan}[a x]}] - (20 - 20i) e^{2i \operatorname{ArcTan}[a x]} \operatorname{Hypergeometric2F1} [1, 1 + i, 2 + i, -e^{2i \operatorname{ArcTan}[a x]}])$$

### Problem 297: Unable to integrate problem.

$$\int \frac{e^{-2 \operatorname{ArcTan}[a x]}}{\sqrt{c + a^2 c x^2}} dx$$

Optimal (type 5, 87 leaves, 3 steps):

$$-\frac{1}{a \sqrt{c + a^2 c x^2}} \left( \frac{2}{5} - \frac{i}{5} \right) 2^{\frac{1}{2}+i} (1 - ia x)^{\frac{1}{2}-i} \sqrt{1 + a^2 x^2} \operatorname{Hypergeometric2F1} \left[ \frac{1}{2} - i, \frac{1}{2} - i, \frac{3}{2} - i, \frac{1}{2} (1 - ia x) \right]$$

Result (type 8, 25 leaves):

$$\int \frac{e^{-2 \operatorname{ArcTan}[a x]}}{\sqrt{c + a^2 c x^2}} dx$$

### Problem 298: Unable to integrate problem.

$$\int \frac{e^{-2 \operatorname{ArcTan}[a x]}}{(c + a^2 c x^2)^{3/2}} dx$$

Optimal (type 3, 38 leaves, 1 step):

$$-\frac{e^{-2 \operatorname{ArcTan}[a x]} (2 - a x)}{5 a c \sqrt{c + a^2 c x^2}}$$

Result (type 8, 25 leaves):

$$\int \frac{e^{-2 \operatorname{ArcTan}[a x]}}{(c + a^2 c x^2)^{3/2}} dx$$

### Problem 299: Unable to integrate problem.

$$\int \frac{e^{-2 \operatorname{ArcTan}[a x]}}{(c + a^2 c x^2)^{5/2}} dx$$

Optimal (type 3, 77 leaves, 2 steps):

$$-\frac{e^{-2 \operatorname{ArcTan}[a x]} (2 - 3 a x)}{13 a c (c + a^2 c x^2)^{3/2}} - \frac{6 e^{-2 \operatorname{ArcTan}[a x]} (2 - a x)}{65 a c^2 \sqrt{c + a^2 c x^2}}$$

Result (type 8, 25 leaves):

$$\int \frac{e^{-2 \operatorname{ArcTan}[a x]}}{(c + a^2 c x^2)^{5/2}} dx$$

### Problem 300: Unable to integrate problem.

$$\int \frac{e^{-2 \operatorname{ArcTan}[a x]}}{(c + a^2 c x^2)^{7/2}} dx$$

Optimal (type 3, 115 leaves, 3 steps):

$$-\frac{e^{-2 \operatorname{ArcTan}[a x]} (2 - 5 a x)}{29 a c (c + a^2 c x^2)^{5/2}} - \frac{20 e^{-2 \operatorname{ArcTan}[a x]} (2 - 3 a x)}{377 a c^2 (c + a^2 c x^2)^{3/2}} - \frac{24 e^{-2 \operatorname{ArcTan}[a x]} (2 - a x)}{377 a c^3 \sqrt{c + a^2 c x^2}}$$

Result (type 8, 25 leaves):

$$\int \frac{e^{-2 \operatorname{ArcTan}[a x]}}{(c + a^2 c x^2)^{7/2}} dx$$

### Problem 314: Result unnecessarily involves higher level functions.

$$\int \frac{e^{i \operatorname{ArcTan}[a x]}}{\sqrt{c + a^2 c x^2}} dx$$

Optimal (type 3, 42 leaves, 3 steps):

$$\frac{i \sqrt{1 + a^2 x^2} \operatorname{Log}[i + a x]}{a \sqrt{c + a^2 c x^2}}$$

Result (type 4, 81 leaves):

$$\left( i \sqrt{1 + a^2 x^2} \left( -2 a \operatorname{EllipticF}[i \operatorname{ArcSinh}[\sqrt{a^2} x], 1] + \sqrt{a^2} \operatorname{Log}[1 + a^2 x^2] \right) \right) / \left( 2 a \sqrt{a^2} \sqrt{c + a^2 c x^2} \right)$$



**Problem 315: Result unnecessarily involves higher level functions.**

$$\int \frac{e^{-i \operatorname{ArcTan}[a x]}}{\sqrt{c + a^2 c x^2}} dx$$

Optimal (type 3, 43 leaves, 3 steps):

$$-\frac{i \sqrt{1 + a^2 x^2} \operatorname{Log}[i - a x]}{a \sqrt{c + a^2 c x^2}}$$

Result (type 4, 81 leaves):

$$-\left( \left( i \sqrt{1 + a^2 x^2} \left( 2 a \operatorname{EllipticF}\left[ i \operatorname{ArcSinh}\left[ \sqrt{a^2} x \right], 1 \right] + \sqrt{a^2} \operatorname{Log}\left[ 1 + a^2 x^2 \right] \right) \right) / \left( 2 a \sqrt{a^2} \sqrt{c + a^2 c x^2} \right) \right)$$

**Problem 337: Result more than twice size of optimal antiderivative.**

$$\int e^{n \operatorname{ArcTan}[a x]} (c + a^2 c x^2)^2 dx$$

Optimal (type 5, 86 leaves, 2 steps):

$$-\frac{1}{a (6 i - n)} 2^{3 - \frac{i n}{2}} c^2 (1 - i a x)^{3 + \frac{i n}{2}} \operatorname{Hypergeometric2F1}\left[-2 + \frac{i n}{2}, 3 + \frac{i n}{2}, 4 + \frac{i n}{2}, \frac{1}{2} (1 - i a x)\right]$$

Result (type 5, 207 leaves):

$$\frac{1}{120 a} c^2 e^{n \operatorname{ArcTan}[a x]} \left( -22 n - n^3 + 120 a x + 22 a n^2 x + a n^4 x - 28 a^2 n x^2 - a^2 n^3 x^2 + 80 a^3 x^3 + 2 a^3 n^2 x^3 - 6 a^4 n x^4 + 24 a^5 x^5 + e^{2 i \operatorname{ArcTan}[a x]} n \left( 32 + 16 i n + 2 n^2 + i n^3 \right) \operatorname{Hypergeometric2F1}\left[1, 1 - \frac{i n}{2}, 2 - \frac{i n}{2}, -e^{2 i \operatorname{ArcTan}[a x]}\right] - i \left( 64 + 20 n^2 + n^4 \right) \operatorname{Hypergeometric2F1}\left[1, -\frac{i n}{2}, 1 - \frac{i n}{2}, -e^{2 i \operatorname{ArcTan}[a x]}\right] \right)$$

**Problem 348: Result more than twice size of optimal antiderivative.**

$$\int e^{n \operatorname{ArcTan}[a x]} (c + a^2 c x^2)^{3/2} dx$$

Optimal (type 5, 121 leaves, 3 steps):

$$-\left( \left( 2^{\frac{5}{2} - \frac{i n}{2}} c (1 - i a x)^{\frac{1}{2} (5 + i n)} \sqrt{c + a^2 c x^2} \operatorname{Hypergeometric2F1}\left[ \frac{1}{2} (-3 + i n), \frac{1}{2} (5 + i n), \frac{1}{2} (7 + i n), \frac{1}{2} (1 - i a x) \right] \right) / \left( a (5 i - n) \sqrt{1 + a^2 x^2} \right) \right)$$

Result (type 5, 267 leaves):

$$\frac{1}{96 a \sqrt{c + a^2 c x^2}} c^2 \left( e^{n \operatorname{ArcTan}[a x]} (1 + a^2 x^2)^2 \left( n - 3 n^3 + 18 a x + 2 a n^2 x + 2 a (-3 + n^2) x \operatorname{Cos}[2 \operatorname{ArcTan}[a x]] - n (-3 + n^2) \sqrt{1 + a^2 x^2} \operatorname{Cos}[3 \operatorname{ArcTan}[a x]] \right) + 8 e^{(i+n) \operatorname{ArcTan}[a x]} (3 i - 3 n - i n^2 + n^3) \sqrt{1 + a^2 x^2} \operatorname{Hypergeometric2F1}\left[1, \frac{1}{2} - \frac{i n}{2}, \frac{3}{2} - \frac{i n}{2}, -e^{2 i \operatorname{ArcTan}[a x]}\right] + 48 e^{n \operatorname{ArcTan}[a x]} (1 + a^2 x^2) \left( -n + a x + (1 + e^{2 i \operatorname{ArcTan}[a x]}) (-i + n) \operatorname{Hypergeometric2F1}\left[1, \frac{1}{2} - \frac{i n}{2}, \frac{3}{2} - \frac{i n}{2}, -e^{2 i \operatorname{ArcTan}[a x]}\right] \right) \right)$$

**Problem 351: Result more than twice size of optimal antiderivative.**

$$\int e^{n \operatorname{ArcTan}[a x]} x^2 (c + a^2 c x^2)^{3/2} dx$$

Optimal (type 5, 283 leaves, 5 steps):

$$\frac{-c n (1 - i a x)^{\frac{1}{2} (5+i n)} (1 + i a x)^{\frac{1}{2} (5-i n)} \sqrt{c + a^2 c x^2}}{30 a^3 \sqrt{1 + a^2 x^2}} + \frac{c x (1 - i a x)^{\frac{1}{2} (5+i n)} (1 + i a x)^{\frac{1}{2} (5-i n)} \sqrt{c + a^2 c x^2}}{6 a^2 \sqrt{1 + a^2 x^2}} + \left( 2^{\frac{3-i n}{2}} c (5 - n^2) (1 - i a x)^{\frac{1}{2} (5+i n)} \sqrt{c + a^2 c x^2} \operatorname{Hypergeometric2F1}\left[\frac{1}{2} (-3 + i n), \frac{1}{2} (5 + i n), \frac{1}{2} (7 + i n), \frac{1}{2} (1 - i a x)\right] \right) / \left( 15 a^3 (5 i - n) \sqrt{1 + a^2 x^2} \right)$$

Result (type 5, 1283 leaves):

$$\left( c^2 \sqrt{1 + a^2 x^2} \left( -\frac{1}{2} e^{n \operatorname{ArcTan}[a x]} (1 + a^2 x^2)^2 \left( \frac{n (-1 + 3 n^2)}{\sqrt{1 + a^2 x^2}} - \frac{2 a x (9 + n^2 + (-3 + n^2) \operatorname{Cos}[2 \operatorname{ArcTan}[a x]])}{\sqrt{1 + a^2 x^2}} + n (-3 + n^2) \operatorname{Cos}[3 \operatorname{ArcTan}[a x]] \right) + 4 e^{(i+n) \operatorname{ArcTan}[a x]} (3 i - 3 n - i n^2 + n^3) \operatorname{Hypergeometric2F1}\left[1, \frac{1}{2} - \frac{i n}{2}, \frac{3}{2} - \frac{i n}{2}, -e^{2 i \operatorname{ArcTan}[a x]}\right] \right) \right) / \left( 48 a^3 \sqrt{c (1 + a^2 x^2)} \right) + \frac{1}{a^3} c^2 \left( -\frac{e^{n \operatorname{ArcTan}[a x]} (19 n - 25 n^3 + n^5) \sqrt{1 + a^2 x^2}}{720 \sqrt{c (1 + a^2 x^2)}} + \left( e^{(i+n) \operatorname{ArcTan}[a x]} (e^{2 i \operatorname{ArcTan}[a x]})^{\frac{1}{2} - \frac{i n}{2} + \frac{1}{2} i (i+n)} (1 + n^2) (45 - 26 n^2 + n^4) \sqrt{1 + a^2 x^2} \operatorname{Hypergeometric2F1}\left[1, -\frac{1}{2} i (i + n), 1 - \frac{1}{2} i (i + n), -e^{2 i \operatorname{ArcTan}[a x]}\right] \right) \right) / \left( 360 (i + n) \right)$$

$$\begin{aligned}
 & \sqrt{c(1+a^2x^2)} + \frac{e^{n \operatorname{ArcTan}[ax]} \sqrt{1+a^2x^2}}{48 \sqrt{c(1+a^2x^2)} \left( \cos\left[\frac{1}{2} \operatorname{ArcTan}[ax]\right] - \sin\left[\frac{1}{2} \operatorname{ArcTan}[ax]\right] \right)^6} + \\
 & \frac{e^{n \operatorname{ArcTan}[ax]} (-30 - 2n + n^2) \sqrt{1+a^2x^2}}{480 \sqrt{c(1+a^2x^2)} \left( \cos\left[\frac{1}{2} \operatorname{ArcTan}[ax]\right] - \sin\left[\frac{1}{2} \operatorname{ArcTan}[ax]\right] \right)^4} + \\
 & \left( \frac{e^{n \operatorname{ArcTan}[ax]} (45 + 26n - 26n^2 - n^3 + n^4) \sqrt{1+a^2x^2}}{1440 \sqrt{c(1+a^2x^2)} \left( \cos\left[\frac{1}{2} \operatorname{ArcTan}[ax]\right] - \sin\left[\frac{1}{2} \operatorname{ArcTan}[ax]\right] \right)^2} - \right. \\
 & \left. \frac{e^{n \operatorname{ArcTan}[ax]} n \sqrt{1+a^2x^2} \sin\left[\frac{1}{2} \operatorname{ArcTan}[ax]\right]}{120 \sqrt{c(1+a^2x^2)} \left( \cos\left[\frac{1}{2} \operatorname{ArcTan}[ax]\right] - \sin\left[\frac{1}{2} \operatorname{ArcTan}[ax]\right] \right)^5} - \right. \\
 & \left. \frac{e^{n \operatorname{ArcTan}[ax]} n (-26 + n^2) \sqrt{1+a^2x^2} \sin\left[\frac{1}{2} \operatorname{ArcTan}[ax]\right]}{720 \sqrt{c(1+a^2x^2)} \left( \cos\left[\frac{1}{2} \operatorname{ArcTan}[ax]\right] - \sin\left[\frac{1}{2} \operatorname{ArcTan}[ax]\right] \right)^3} - \right. \\
 & \left. \frac{e^{n \operatorname{ArcTan}[ax]} n (19 - 25n^2 + n^4) \sqrt{1+a^2x^2} \sin\left[\frac{1}{2} \operatorname{ArcTan}[ax]\right]}{720 \sqrt{c(1+a^2x^2)} \left( \cos\left[\frac{1}{2} \operatorname{ArcTan}[ax]\right] - \sin\left[\frac{1}{2} \operatorname{ArcTan}[ax]\right] \right)} - \right. \\
 & \left. \frac{e^{n \operatorname{ArcTan}[ax]} \sqrt{1+a^2x^2}}{48 \sqrt{c(1+a^2x^2)} \left( \cos\left[\frac{1}{2} \operatorname{ArcTan}[ax]\right] + \sin\left[\frac{1}{2} \operatorname{ArcTan}[ax]\right] \right)^6} + \right. \\
 & \left. \frac{e^{n \operatorname{ArcTan}[ax]} n \sqrt{1+a^2x^2} \sin\left[\frac{1}{2} \operatorname{ArcTan}[ax]\right]}{120 \sqrt{c(1+a^2x^2)} \left( \cos\left[\frac{1}{2} \operatorname{ArcTan}[ax]\right] + \sin\left[\frac{1}{2} \operatorname{ArcTan}[ax]\right] \right)^5} - \right. \\
 & \left. \frac{e^{n \operatorname{ArcTan}[ax]} (-30 + 2n + n^2) \sqrt{1+a^2x^2}}{480 \sqrt{c(1+a^2x^2)} \left( \cos\left[\frac{1}{2} \operatorname{ArcTan}[ax]\right] + \sin\left[\frac{1}{2} \operatorname{ArcTan}[ax]\right] \right)^4} + \right. \\
 & \left. \frac{e^{n \operatorname{ArcTan}[ax]} n (-26 + n^2) \sqrt{1+a^2x^2} \sin\left[\frac{1}{2} \operatorname{ArcTan}[ax]\right]}{720 \sqrt{c(1+a^2x^2)} \left( \cos\left[\frac{1}{2} \operatorname{ArcTan}[ax]\right] + \sin\left[\frac{1}{2} \operatorname{ArcTan}[ax]\right] \right)^3} - \right. \\
 & \left. \frac{e^{n \operatorname{ArcTan}[ax]} (45 - 26n - 26n^2 + n^3 + n^4) \sqrt{1+a^2x^2}}{1440 \sqrt{c(1+a^2x^2)} \left( \cos\left[\frac{1}{2} \operatorname{ArcTan}[ax]\right] + \sin\left[\frac{1}{2} \operatorname{ArcTan}[ax]\right] \right)^2} + \right. \\
 & \left. \frac{e^{n \operatorname{ArcTan}[ax]} n (19 - 25n^2 + n^4) \sqrt{1+a^2x^2} \sin\left[\frac{1}{2} \operatorname{ArcTan}[ax]\right]}{720 \sqrt{c(1+a^2x^2)} \left( \cos\left[\frac{1}{2} \operatorname{ArcTan}[ax]\right] + \sin\left[\frac{1}{2} \operatorname{ArcTan}[ax]\right] \right)} \right)
 \end{aligned}$$

Problem 360: Unable to integrate problem.

$$\int e^{n \operatorname{ArcTan}[ax]} (c + a^2 c x^2)^{1/3} dx$$

Optimal (type 5, 120 leaves, 3 steps):

$$-\left(\left(3 \times 2^{\frac{4-i n}{3}-\frac{i n}{2}}(1-i a x)^{\frac{1}{6}(8+3 i n)}(c+a^2 c x^2)^{1/3} \text{Hypergeometric2F1}\left[\frac{1}{6}(-2+3 i n), \frac{1}{6}(8+3 i n), \frac{1}{6}(14+3 i n), \frac{1}{2}(1-i a x)\right]\right) / \left(a(8 i-3 n)(1+a^2 x^2)^{1/3}\right)\right)$$

Result (type 8, 25 leaves):

$$\int e^{n \text{ArcTan}[a x]}(c+a^2 c x^2)^{1/3} dx$$

**Problem 361: Unable to integrate problem.**

$$\int \frac{e^{n \text{ArcTan}[a x]}}{(c+a^2 c x^2)^{1/3}} dx$$

Optimal (type 5, 120 leaves, 3 steps):

$$-\left(\left(3 \times 2^{\frac{2-i n}{3}-\frac{i n}{2}}(1-i a x)^{\frac{1}{6}(4+3 i n)}(1+a^2 x^2)^{1/3} \text{Hypergeometric2F1}\left[\frac{1}{6}(2+3 i n), \frac{1}{6}(4+3 i n), \frac{1}{6}(10+3 i n), \frac{1}{2}(1-i a x)\right]\right) / \left(a(4 i-3 n)(c+a^2 c x^2)^{1/3}\right)\right)$$

Result (type 8, 25 leaves):

$$\int \frac{e^{n \text{ArcTan}[a x]}}{(c+a^2 c x^2)^{1/3}} dx$$

**Problem 362: Unable to integrate problem.**

$$\int \frac{e^{n \text{ArcTan}[a x]}}{(c+a^2 c x^2)^{2/3}} dx$$

Optimal (type 5, 120 leaves, 3 steps):

$$-\left(\left(3 \times 2^{\frac{1-i n}{3}-\frac{i n}{2}}(1-i a x)^{\frac{1}{6}(2+3 i n)}(1+a^2 x^2)^{2/3} \text{Hypergeometric2F1}\left[\frac{1}{6}(2+3 i n), \frac{1}{6}(4+3 i n), \frac{1}{6}(8+3 i n), \frac{1}{2}(1-i a x)\right]\right) / \left(a(2 i-3 n)(c+a^2 c x^2)^{2/3}\right)\right)$$

Result (type 8, 25 leaves):

$$\int \frac{e^{n \text{ArcTan}[a x]}}{(c+a^2 c x^2)^{2/3}} dx$$

**Problem 363: Unable to integrate problem.**

$$\int \frac{e^{n \text{ArcTan}[a x]}}{(c+a^2 c x^2)^{4/3}} dx$$

Optimal (type 5, 123 leaves, 3 steps):

$$\left( 3 \times 2^{-\frac{1}{3} - \frac{i n}{2}} (1 - i a x)^{\frac{1}{6} (-2 + 3 i n)} (1 + a^2 x^2)^{1/3} \text{Hypergeometric2F1} \left[ \frac{1}{6} (-2 + 3 i n), \frac{1}{6} (8 + 3 i n), \frac{1}{6} (4 + 3 i n), \frac{1}{2} (1 - i a x) \right] \right) / \left( a c (2 i + 3 n) (c + a^2 c x^2)^{1/3} \right)$$

Result (type 8, 25 leaves):

$$\int \frac{e^{n \text{ArcTan}[a x]}}{(c + a^2 c x^2)^{4/3}} dx$$

**Problem 364: Unable to integrate problem.**

$$\int e^{n \text{ArcTan}[a x]} x^m (c + a^2 c x^2) dx$$

Optimal (type 6, 49 leaves, 2 steps):

$$\frac{c x^{1+m} \text{AppellF1} \left[ 1 + m, -1 - \frac{i n}{2}, -1 + \frac{i n}{2}, 2 + m, i a x, -i a x \right]}{1 + m}$$

Result (type 8, 24 leaves):

$$\int e^{n \text{ArcTan}[a x]} x^m (c + a^2 c x^2) dx$$

**Problem 366: Unable to integrate problem.**

$$\int \frac{e^{n \text{ArcTan}[a x]} x^m}{(c + a^2 c x^2)^2} dx$$

Optimal (type 6, 51 leaves, 2 steps):

$$\frac{x^{1+m} \text{AppellF1} \left[ 1 + m, 2 - \frac{i n}{2}, 2 + \frac{i n}{2}, 2 + m, i a x, -i a x \right]}{c^2 (1 + m)}$$

Result (type 8, 26 leaves):

$$\int \frac{e^{n \text{ArcTan}[a x]} x^m}{(c + a^2 c x^2)^2} dx$$

**Problem 367: Unable to integrate problem.**

$$\int \frac{e^{n \text{ArcTan}[a x]} x^m}{(c + a^2 c x^2)^3} dx$$

Optimal (type 6, 51 leaves, 2 steps):

$$\frac{x^{1+m} \text{AppellF1} \left[ 1 + m, 3 - \frac{i n}{2}, 3 + \frac{i n}{2}, 2 + m, i a x, -i a x \right]}{c^3 (1 + m)}$$

Result (type 8, 26 leaves):

$$\int \frac{e^{n \operatorname{ArcTan}[a x]} x^m}{(c + a^2 c x^2)^3} dx$$

**Problem 368: Unable to integrate problem.**

$$\int \frac{e^{n \operatorname{ArcTan}[a x]} x^m}{\sqrt{c + a^2 c x^2}} dx$$

Optimal (type 6, 79 leaves, 3 steps):

$$\left( x^{1+m} \sqrt{1+a^2 x^2} \operatorname{AppellF1}\left[1+m, \frac{1}{2}(1-in), \frac{1}{2}(1+in), 2+m, i a x, -i a x\right] \right) / \left( (1+m) \sqrt{c+a^2 c x^2} \right)$$

Result (type 8, 28 leaves):

$$\int \frac{e^{n \operatorname{ArcTan}[a x]} x^m}{\sqrt{c + a^2 c x^2}} dx$$

**Problem 369: Unable to integrate problem.**

$$\int \frac{e^{n \operatorname{ArcTan}[a x]} x^m}{(c + a^2 c x^2)^{3/2}} dx$$

Optimal (type 6, 82 leaves, 3 steps):

$$\left( x^{1+m} \sqrt{1+a^2 x^2} \operatorname{AppellF1}\left[1+m, \frac{1}{2}(3-in), \frac{1}{2}(3+in), 2+m, i a x, -i a x\right] \right) / \left( c(1+m) \sqrt{c+a^2 c x^2} \right)$$

Result (type 8, 28 leaves):

$$\int \frac{e^{n \operatorname{ArcTan}[a x]} x^m}{(c + a^2 c x^2)^{3/2}} dx$$

**Problem 370: Unable to integrate problem.**

$$\int \frac{e^{n \operatorname{ArcTan}[a x]} x^m}{(c + a^2 c x^2)^{5/2}} dx$$

Optimal (type 6, 82 leaves, 3 steps):

$$\left( x^{1+m} \sqrt{1+a^2 x^2} \operatorname{AppellF1}\left[1+m, \frac{1}{2}(5-in), \frac{1}{2}(5+in), 2+m, i a x, -i a x\right] \right) / \left( c^2(1+m) \sqrt{c+a^2 c x^2} \right)$$

Result (type 8, 28 leaves):

$$\int \frac{e^{n \operatorname{ArcTan}[a x]} x^m}{(c + a^2 c x^2)^{5/2}} dx$$

Problem 371: Unable to integrate problem.

$$\int e^{n \operatorname{ArcTan}[a x]} (c + a^2 c x^2)^p dx$$

Optimal (type 5, 115 leaves, 3 steps):

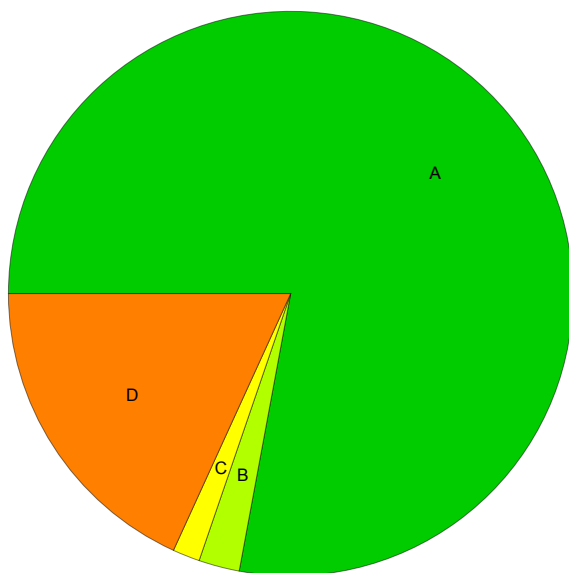
$$\left( 2^{1 - \frac{i n}{2} + p} (1 - i a x)^{1 + \frac{i n}{2} + p} (1 + a^2 x^2)^{-p} (c + a^2 c x^2)^p \operatorname{Hypergeometric2F1}\left[\frac{i n}{2} - p, 1 + \frac{i n}{2} + p, 2 + \frac{i n}{2} + p, \frac{1}{2} (1 - i a x)\right] \right) / (a (n - 2 i (1 + p)))$$

Result (type 8, 23 leaves):

$$\int e^{n \operatorname{ArcTan}[a x]} (c + a^2 c x^2)^p dx$$

## Summary of Integration Test Results

385 integration problems



A - 300 optimal antiderivatives

B - 9 more than twice size of optimal antiderivatives

C - 6 unnecessarily complex antiderivatives

D - 70 unable to integrate problems

E - 0 integration timeouts